Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

June 2017

Manitoba

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This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

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General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Training in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION				
Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error). **Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).**				
E1 ●	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ●	**E8** ●	**E9** ○	**E10** ○

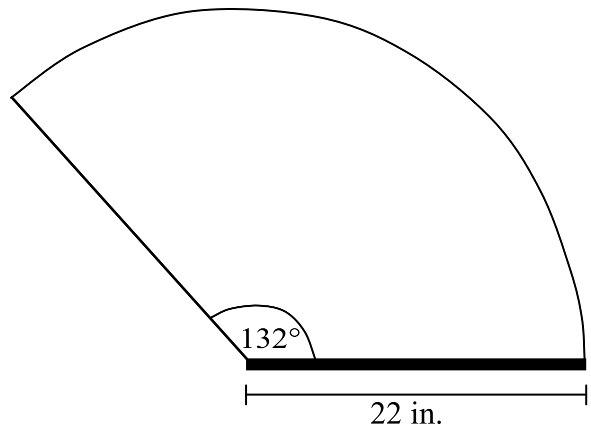
Example: Marks assigned to the student.

	Booklet 1	Selected Response	Booklet 2	Communication Errors (Deduct)	Total
Marks Awarded	25	7	40	1½	70½
Total Marks	**36**	**9**	**45**	**maximum deduction of 5 marks**	**90**

Scoring Guidelines for Booklet 1 Questions

Question 1 T1

A section of a car windshield is cleaned by a wiper as shown in the diagram below. The arm of the wiper is 22 inches, and it rotates through a central angle of $132°$. Determine the length of the arc that is created by the tip of the wiper.



Solution

$\theta = 132 \times \frac{\pi}{180}$ 1 mark for conversion

$= \frac{132\pi}{180}$ **or** $\frac{11\pi}{15}$

$s = \theta r$

$s = \frac{11\pi}{15}(22)$ 1 mark for substitution

$s = \frac{242\pi}{15}$ inches

2 marks

or

$s = 50.684$ inches

Exemplar 1

$$132 \times \frac{\pi}{180} = \frac{11\pi}{15}$$

$$d = 22 \qquad r = \frac{22}{2}$$

$$S = \theta r$$

$$S = \left(\frac{11\pi}{15}\right)(11)$$

$$S = 25.342$$

1½ out of 2

award full marks
− ½ mark for procedural error of incorrect radius
E5 (units of measure omitted in final answer)

Exemplar 2

$$132^\circ \times \frac{\pi}{180} = 2.3$$

$$S = r\theta$$

$$S = (22)(2.3)$$

$$S = 50.6 \text{ in.}$$

2 out of 2

award full marks
E6 (rounding too early)

Exemplar 3

$$S = \theta r$$

$$S = 132(22)$$

$$S = 2904 \text{ arc length}$$

1 out of 2

+ 1 mark for substitution
E5 (units of measure omitted in final answer)

Question 2 P3

There are 20 boys and 11 girls who can be selected to be on a team.

Determine the number of ways that 7 boys and 5 girls can be selected for this team.

Solution

$_{20}C_7 \cdot {}_{11}C_5 = 35\ 814\ 240$

½ mark for $_{20}C_7$

½ mark for $_{11}C_5$

1 mark for the product of combinations

2 marks

Exemplar 1

$$20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 2.166 \times 10^{13}$$

1 out of 2

award full marks
− 1 mark for concept error (permutations instead of combinations)

Exemplar 2

Choose 7 boys — Choose 5 girls

$${}_{20}C_7 \times {}_{11}C_5$$

$$= \frac{20!}{(20-7)!\,7!} \qquad = \frac{11!}{(11-5)!\,5!}$$

$$= \frac{20!}{13!\,7!} \qquad = \frac{11!}{6!\,5!}$$

$$= 77520 \qquad = 465$$

$$77520 \times 465$$

$$= 36046800 \text{ ways}$$

1½ out of 2

award full marks
− ½ mark for arithmetic error in line 4

Exemplar 3

$${}_{20}C_7 + {}_{11}C_5$$

$$77520 + 462$$

$= 77982$ ways 7 boys and 5 girls can be selected.

1 out of 2

+ ½ mark for ${}_{20}C_7$

+ ½ mark for ${}_{11}C_5$

A water filtration system which removes impurities from a sample of water can be modelled by $P = 0.25(0.55)^n$, where:

$P =$ the percentage of impurities remaining, in decimal form

$n =$ the number of filters

Determine, algebraically, how many filters are required so that less than 1% of the impurities remain in the water sample. Express your answer as a whole number.

Solution

$0.01 = 0.25(0.55)^n$	½ mark for substitution
$0.04 = (0.55)^n$	
$\log(0.04) = n\log(0.55)$	½ mark for applying logarithms ½ mark for power law
$\dfrac{\log(0.04)}{\log(0.55)} = n$	
$n = 5.384\ 203$	½ mark for solving for n
$\therefore$ 6 filters are needed	**2 marks**

Exemplar 1

$$P = 0.25(0.55)^n$$

$$1 = 0.25(0.55)^n$$

$$\frac{1}{0.25} = (0.55)^n$$

$$4 = 0.55^n$$

$$\log 4 = \log 0.55^n$$

$$\frac{\log 4}{\log 0.55} = \frac{n(\log 0.55)}{\log 0.55}$$

$$-2.318 = n$$

1½ out of 2

+ ½ mark for applying logarithms
+ ½ mark for power law
+ ½ mark for solving for *n*
E6 rounding error (answer not expressed as a whole number)

Exemplar 2

$$0.01 = 0.25(0.55)^n$$

$$\log 0.01 = \log 0.25(0.55)^n$$

$$\log 0.01 = n \log (0.25)(0.55)$$

$$\log 0.01 = n \log 0.1375$$

$$\frac{\log 0.01}{\log 0.1375} = n$$

$$n = 2.321$$

2 filters needed

---

1 out of 2

award full marks

−1 mark for concept error in line 3

E6 rounding error (not rounding up)

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In the binomial expansion of $\left(x^2 - \frac{2}{y}\right)^8$, determine the middle term in simplified form.

Solution

$t_5 = {}_8C_4\left(x^2\right)^{8-4}\left(-\frac{2}{y}\right)^4$ 2 marks (1 mark for ${}_8C_4$, ½ mark for each consistent factor)

$t_5 = 70x^8\left(\frac{16}{y^4}\right)$

$t_5 = \frac{1120x^8}{y^4}$ 1 mark for simplification (½ mark for coefficient, ½ mark for exponents)

3 marks

Exemplar 1

$$t_{4+1} = {}_8C_4 \, (x^2)^{8-4} \left(-\frac{2}{y}\right)^4$$

$$t_5 = 70 \, (x^2)^4 \left(-\frac{16}{y^4}\right)$$

$$t_5 = 70 \, (x^8) \left(-\frac{16}{y^4}\right)$$

$$t_5 = -1120 \, x^8 y^4$$

2 out of 3

+ 1 mark for ${}_8C_4$

+ 1 mark for consistent factors

Exemplar 2

0 1 2 3 (4) 5 6 7 8

_ _ _ _ _ _ _ _ _

↑ 1 middle term

$n = 8$
$k = 3$

$$t_{3+1} = {}_8C_3(x^2)^{8-3}\left(\frac{-2}{y}\right)^3$$

$$t_4 = 56(x^2)^5\left(\frac{-8}{y^3}\right)$$

$$t_4 = 56(x^{10})\left(\frac{-8}{y^3}\right)$$

$$t_4 = 56x^{10}\left(\frac{-8}{y^3}\right)$$

$$\boxed{t_4 = \frac{-448x^{10}}{y^3}}$$

2 out of 3

+ 1 mark for consistent factors
+ 1 mark for simplification

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Solve the following equation algebraically over the interval $[0, 2\pi]$.

$$6\sin^2\theta + \sin\theta - 1 = 0$$

Solution

$(3\sin\theta - 1)(2\sin\theta + 1) = 0$

$3\sin\theta - 1 = 0$	$2\sin\theta + 1 = 0$	
$\sin\theta = \frac{1}{3}$	$\sin\theta = -\frac{1}{2}$	1 mark for solving for $\sin\theta$ (½ mark for each branch)
$\theta_r = 0.339836$		
$\theta = 0.340$	$\theta = \frac{7\pi}{6}$	
$\theta = 2.802$	$\theta = \frac{11\pi}{6}$	2 marks (½ mark for each value of θ)

or

$\theta = 0.340,\ 2.802,\ 3.665,\ 5.760$ **3 marks**

Exemplar 1

$$6\sin^2\theta + \sin\theta - 1 = 0$$

$$6\sin^2\theta - 2\sin\theta + 3\sin\theta - 1$$

$$2\sin(3\sin\theta - 1) + 1(3\sin\theta - 1)$$

$$(2\sin\theta + 1)(3\sin\theta - 1)$$

$\sin\theta = \frac{1}{2}$ | $\sin\theta = -\frac{1}{3}$

$\theta_R = 30^\circ$

$\theta = 30^\circ, 150^\circ$

1½ out of 3

+ 1 mark for solving for $\sin\theta$
+ 1 mark for consistent values of θ
− ½ mark for arithmetic error in line 5
E2 (changing an equation to an expression in lines 2 to 4)
E3 (variable omitted in line 3)
E5 (answer stated in degrees instead of radians)

Exemplar 2

$$6\sin^2\theta + \sin\theta - 1 = 0$$

$$(3\sin\theta + 1)(2\sin\theta - 1) = 0$$

$$\sin\theta = \frac{-1}{3} \quad \sin\theta = \frac{1}{2}$$

$$\sin^{-1}\left(\frac{1}{3}\right) = 0.339836$$

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

Quad III

$$\theta = \pi + \theta r$$
$$= \pi + 0.339836$$
$$= 3.481428$$

Quad IV

$$\theta = 2\pi - \theta r$$
$$= 2\pi - 0.339836$$
$$= 5.943349$$

Quad I

$$\theta = \theta r$$
$$= \frac{\pi}{6}$$

Quad II

$$\theta = \pi - \theta r$$
$$= \pi - \frac{\pi}{6}$$
$$= \frac{5\pi}{6}$$

$$\left\{ \theta = \frac{\pi}{6}, \frac{5\pi}{6}, 3.481, 5.943 \right\}$$

2½ out of 3

award full marks

−½ mark for arithmetic error in line 2

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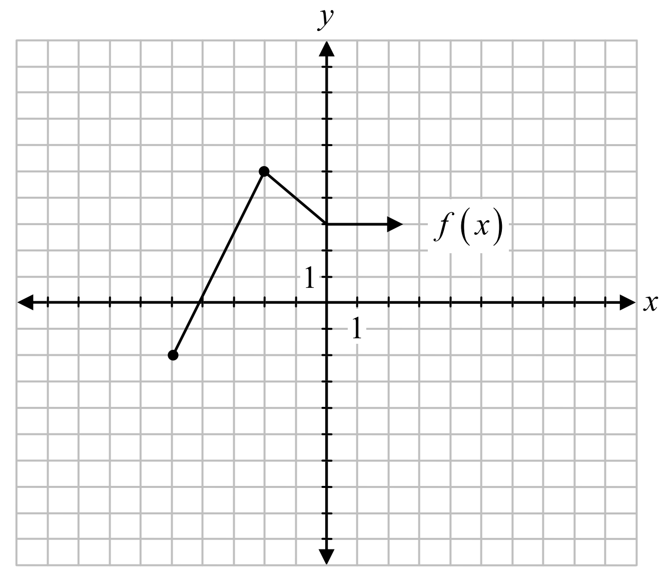
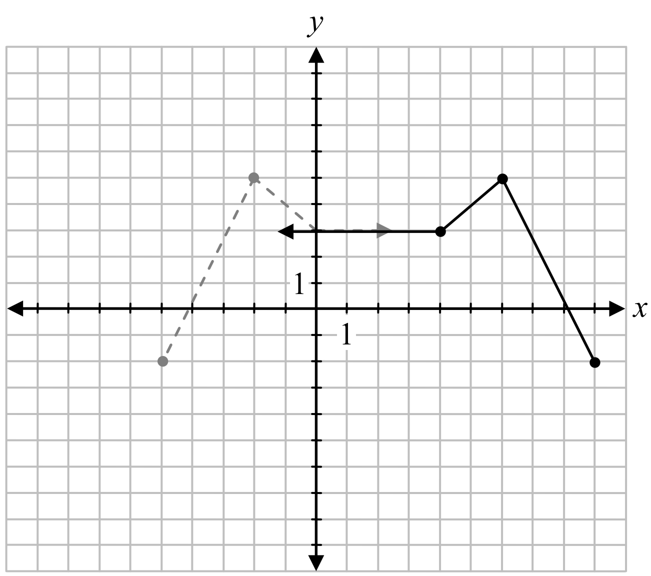
Given the graph of $y = f(x)$, sketch the graph of $y = f(-x + 4)$.



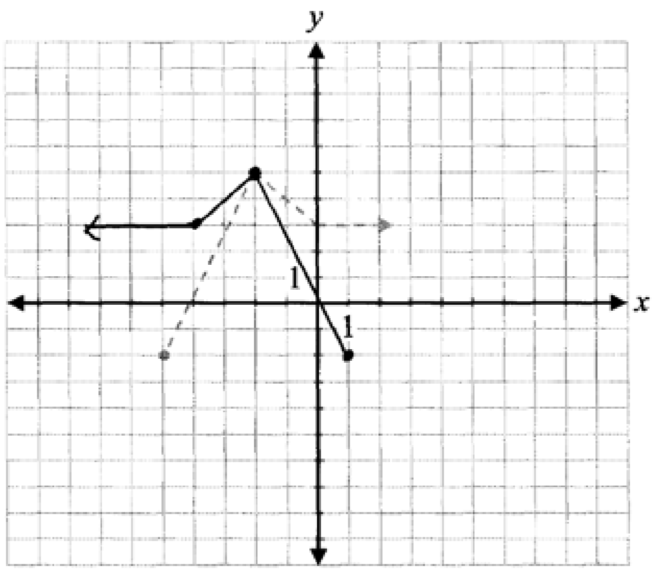
Solution



1 mark for reflection over the y-axis
1 mark for horizontal translation

2 marks

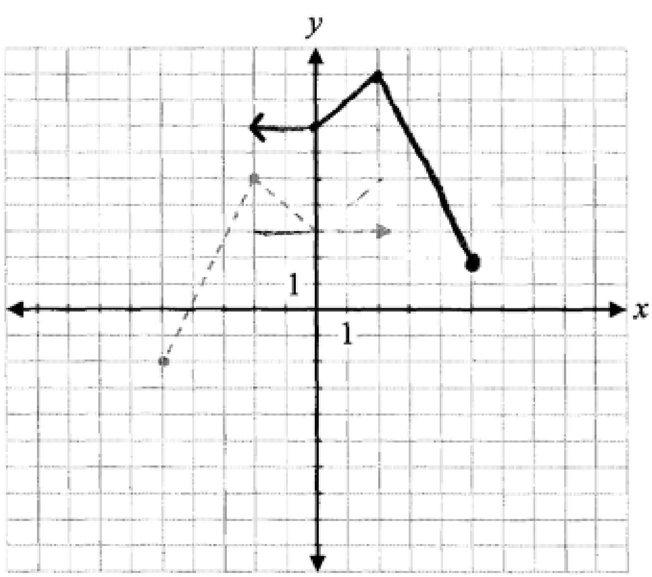
Exemplar 1



1 out of 2

+ 1 mark for reflection over the *y*-axis

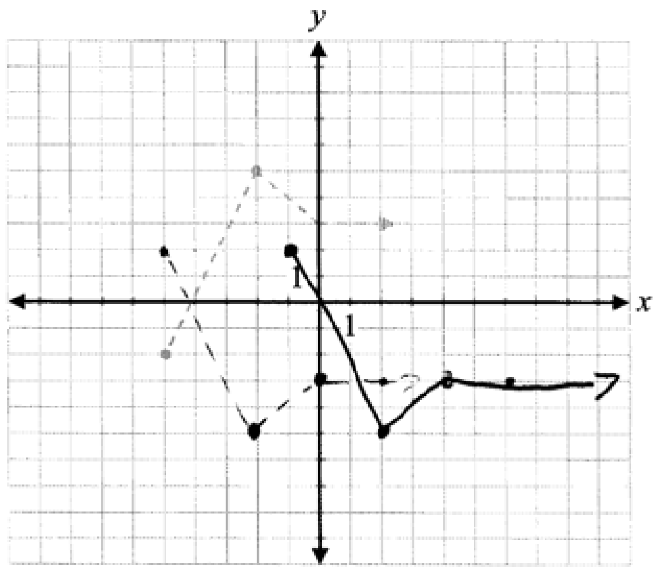
Exemplar 2



1 out of 2

+ 1 mark for reflection over the *y*-axis

Exemplar 3



1 out of 2

+ 1 mark for horizontal translation

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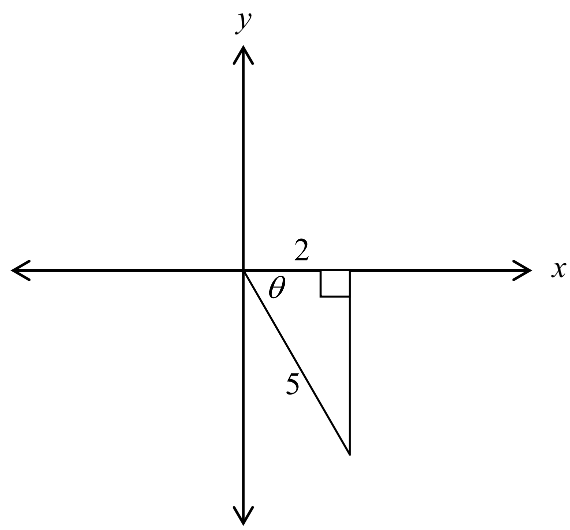
Question 7 T2, T3

Given the following triangle, determine $\csc\theta$.



Solution

$x^2 + y^2 = r^2$

$2^2 + y^2 = 5^2$ ½ mark for substitution

$y^2 = 21$

$y = \pm\sqrt{21}$ ½ mark for solving for y

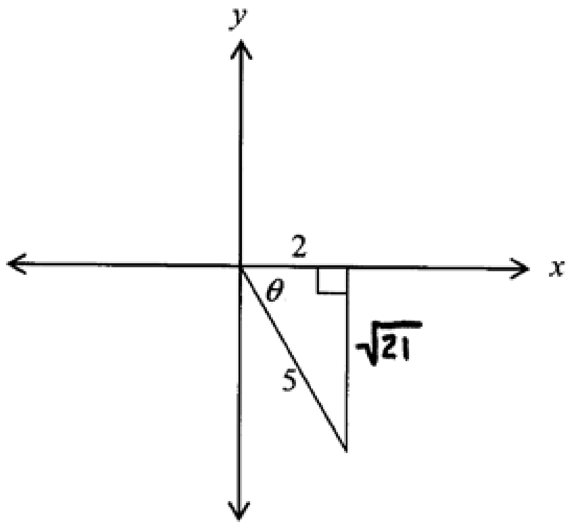
$\csc\theta = -\frac{5}{\sqrt{21}}$ 1 mark for $\csc\theta$ (½ mark for quadrant, ½ mark for value)

2 marks

Note(s):

- Accept any of the following values for y : $y = \pm\sqrt{21}$, $y = \sqrt{21}$, or $y = -\sqrt{21}$.

Exemplar 1



$5^2 - 2^2 = Opp^2$

$25 - 4 = Opp^2$

$21 = Opp^2$

$\sqrt{21} = opp$

$\sin\theta = \frac{\sqrt{21}}{5}$

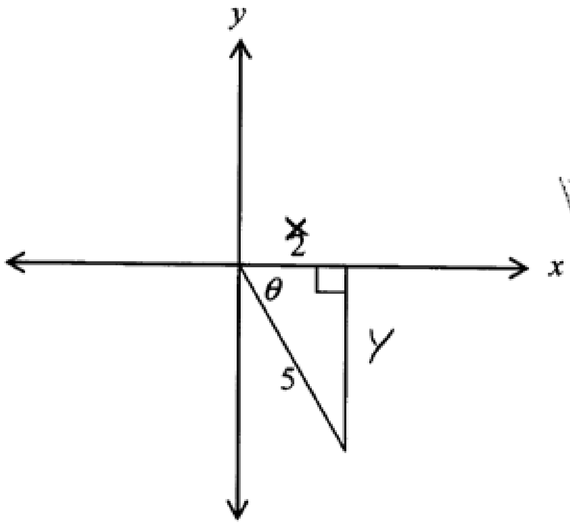
$\csc\theta = \frac{1}{\sin\theta}$

$\therefore \csc\theta = \frac{5}{\sqrt{21}}$

1½ out of 2

+ ½ mark for substitution
+ ½ mark for solving for y
+ ½ mark for value of $\csc\theta$

Exemplar 2



$$\frac{1}{-\sqrt{21}} = \csc\theta$$

$$\sin\theta = y$$

$$y^2 = 5^2 - 2^2$$

$$y^2 = 25 - 4$$

$$y^2 = 21$$

$$y = -\sqrt{21}$$

1½ out of 2

+ ½ mark for substitution
+ ½ mark for solving for y
+ ½ mark for quadrant of $\csc\theta$

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Question 8 P2

Solve algebraically.

$${}_nP_2 = 9n$$

Solution

$$\frac{n!}{(n-2)!} = 9n$$ ½ mark for substitution into equation

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 9n$$ 1 mark for factorial expansion

$$n(n-1) = 9n$$ ½ mark for simplification of factorials

$$n^2 - n = 9n$$

$$n^2 - 10n = 0$$

$$n(n-10) = 0$$

$$\cancel{n = 0} \quad n = 10$$

½ mark for rejecting the extraneous root
½ mark for the value of n

3 marks

Exemplar 1

$$\frac{n!}{(n-2)!} = 9n$$

$$\frac{(n)(n-1)(n-2)!}{(n-2)!} = 9n$$

$$\frac{(n)(n-1)}{n} = \frac{9n}{n}$$

$$n-1 = 9$$

$$\boxed{n = 10}$$

2½ out of 3

+ ½ mark for substitution
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
+ ½ mark for the value of *n*

Exemplar 2

$$\frac{n!}{(n-2)!} = 9n$$

$$\frac{(n)(n-1)(n-2)!}{(n-2)!} = 9n$$

$$n^2 - n = 9n$$

$$n^2 + 8n = 0$$

$$n(n+8) = 0$$

~~$n=0$~~ $n = -8$

2 out of 3

+ ½ mark for substitution
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials

Exemplar 3

$$\frac{n!}{(n-2)!} = 9n \qquad n \geq 2$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 9n$$

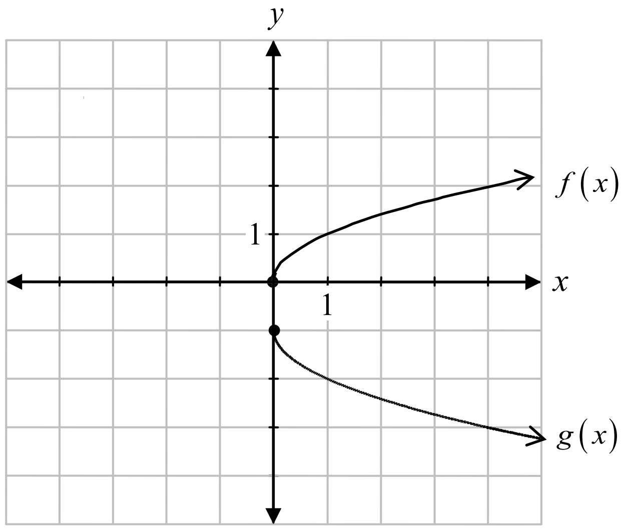
$$\frac{\cancel{n}(n-1)}{\cancel{n}} = \frac{9\cancel{n}}{\cancel{n}}$$

$$n-1 = 9$$

$$\boxed{n = 10}$$

3 out of 3

Describe the transformations applied to the graph of $f(x)$ to obtain the graph of $g(x)$.



Solution

A reflection over the x-axis then a vertical translation of one unit down.

or

A vertical translation of one unit up then a reflection over the x-axis.

1 mark for reflection over the x-axis
1 mark for vertical translation

2 marks

Note(s):

- Award a maximum of 1 mark if correct transformations are given in the incorrect order.

Exemplar 1

$-f(x)-1$

0 out of 2

Exemplar 2

1. There is a translation of one unit down
2. There is a reflection over the x axis (the "a" in the formula will be negative)

$g(x) = -\sqrt{x} - 1$

1 out of 2

award full marks
− 1 mark for concept error (incorrect order)

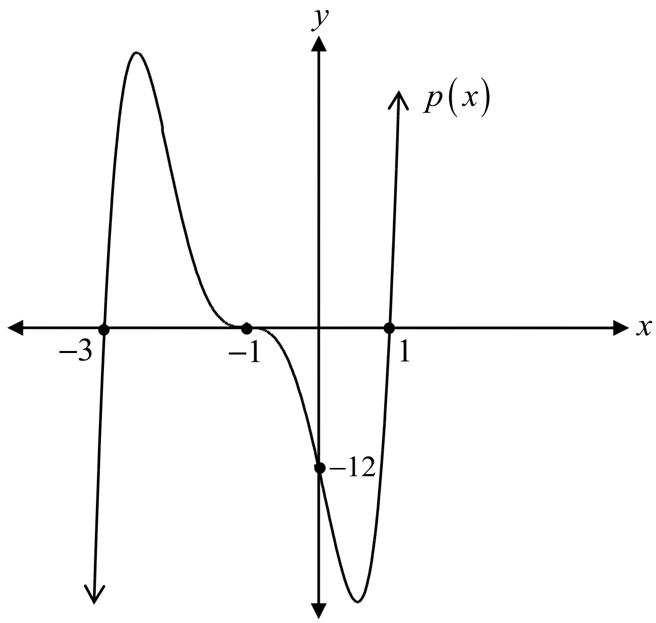
Exemplar 3

The graph of g(x) has a reflection over the "y" axis and also the graph moves one unit down.

½ out of 2

+ 1 mark for vertical translation
− ½ mark for lack of clarity in description

Determine, algebraically, the value of the leading coefficient of the graph of the polynomial function, $p(x)$.



Solution

$p(x) = a(x+3)(x+1)^3(x-1)$ — ½ mark for factors of $p(x)$; ½ mark for odd multiplicity for $(x+1)$ greater than 1

$-12 = a(3)(1)^3(-1)$ — ½ mark for substituting $p(0) = -12$

$-12 = -3a$

$a = 4$ — ½ mark for value of a

2 marks

Exemplar 1

$$p(x) = a(x+3)(x+1)(x-1)$$

$$y = a(x+3)(x+1)(x-1)$$

$$-12 = a(0+3)(0+1)(0-1)$$

$$-12 = a(3)(1)(-1)$$

$$\frac{-12}{-3} = \frac{-3a}{-3}$$

$$\boxed{4 = a}$$

1½ out of 2

+ ½ mark for factors of $p(x)$
+ ½ mark for substituting $p(0) = -12$
+ ½ mark for value of a

Exemplar 2

$$(x+3)(x+1)(x-1) = 0$$

$$(x+3)(x^2 - x + x - 1) = 0$$

$$(x+3)(x^2 - 1) = 0$$

$$(x^3 - x + 3x^2 - 3) = 0$$

$$x^3 + 3x^2 - x - 3 = 0$$

The value of the leading coefficient is 1

½ out of 2

+ ½ mark for factors of $p(x)$

Exemplar 3

$(x+3)(x+1)^3(x-1)$

the leading coefficient would be 4.

1½ out of 2

+ ½ mark for factors of $p(x)$
+ ½ mark for odd multiplicity for $(x+1)$ greater than 1
+ ½ mark for value of a

Exemplar 4

$(x+3)(x+1)^5(x-1)$

1 out of 2

+ ½ mark for factors of $p(x)$
+ ½ mark for odd multiplicity for $(x+1)$ greater than 1

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Question 11 P1

Frank, Liam, Chan, and Thao are going to a movie.

Determine the number of ways they can sit in a row of four chairs, if Frank and Chan must sit beside each other.

Solution

$3!2!$	½ mark for $3!$ ½ mark for $2!$ as a product of factorials
12	**1 mark**

Exemplar 1

$2! = 2$

0 out of 1

Exemplar 2

$3!\,2! = \boxed{6 \text{ ways}}$

½ out of 1

award full marks
− ½ mark for arithmetic error

Exemplar 3

$3!\,2!$

1 out of 1

award full marks
E1 (final answer not stated)

Exemplar 4

$3! = 6$

½ out of 1

\+ ½ mark for 3!

Determine, algebraically, if $f(x) = \dfrac{1}{x+5}$ and $g(x) = \dfrac{1}{x-5}$ are inverses of each other.

Justify your answer.

Solution

Method 1

Let $f(x) = y$

$$y = \frac{1}{x+5}$$

$$x = \frac{1}{y+5}$$ 1 mark for switching x and y values

$$y + 5 = \frac{1}{x}$$

$$y = \frac{1}{x} - 5$$ ½ mark for solving for y

$$f^{-1}(x) = \frac{1}{x} - 5$$

$\therefore\ f^{-1}(x) \neq g(x)$ ½ mark for justification

2 marks

Method 2

1 mark for $f(g(x))$ or $g(f(x))$

$$f(g(x)) = \frac{1}{\left(\frac{1}{x-5}\right) + 5} = \frac{1}{\frac{1 + 5x - 25}{x-5}} = \frac{1}{\frac{5x - 24}{x-5}} = \frac{x-5}{5x-24}$$

½ mark for simplification

$$g(f(x)) = \frac{1}{\left(\frac{1}{x+5}\right) - 5} = \frac{1}{\frac{1 - 5x - 25}{x+5}} = \frac{1}{\frac{-5x - 24}{x+5}} = \frac{x+5}{-5x-24}$$

$\therefore$ They are not inverses because $f(g(x)) \neq x$ or $g(f(x)) \neq x$. ½ mark for justification

2 marks

Exemplar 1

Plug in for x

$$f(g(x)) = \frac{1}{\left(\frac{1}{x-5}\right)+5}$$

$$g(f(x)) = \frac{1}{\left(\frac{1}{x+5}\right)-5}$$

1 out of 2

+ 1 mark for $f\left(g\left(x\right)\right)$ or $g\left(f\left(x\right)\right)$

Exemplar 2

$f(x) = \frac{1}{x+5}$

$y = \frac{1}{x+5}$

$x = \frac{1}{y+5}$

$xy+5 = 1$

$y+5 = \frac{1}{x}$

$y = \frac{1}{x-5} = g(x)$

$g(x) = \frac{1}{x-5}$

$y = \frac{1}{x-5}$

$x = \frac{1}{y-5}$

$xy-5 = 1$

$y-5 = \frac{1}{x}$

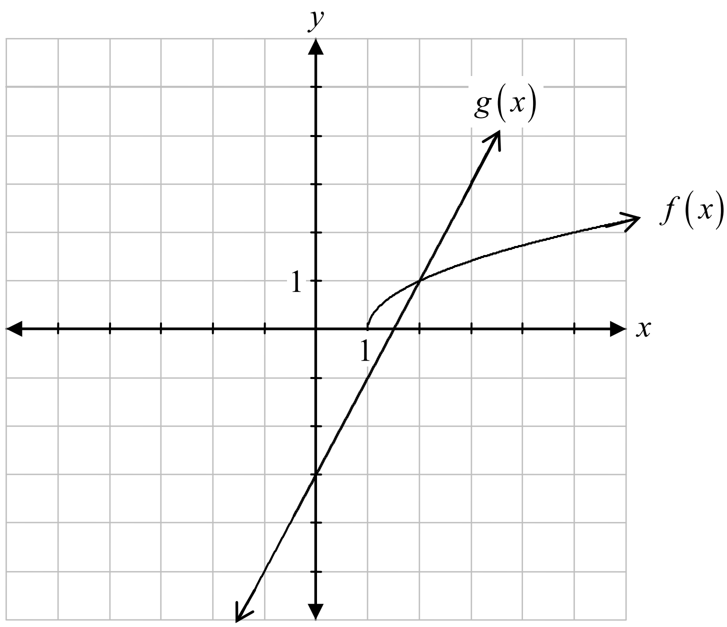
$y = \frac{1}{x+5} = f(x)$

$\therefore$ the function are inverses of each other

1½ out of 2

award full marks
− ½ mark for arithmetic error in line 6
E4 (missing brackets but still implied in line 4)

Using the graphs of $y = f(x)$ and $y = g(x)$, solve $f(x) = g(x)$.



Solution

$x = 2$

1 mark

Exemplar 1

$x = 2$

$y = 1$

0 out of 1

award full marks
-1 mark for concept error (including the *y*-value)

Exemplar 2

$(2, 1)$

0 out of 1

award full marks
-1 mark for concept error (including the *y*-value)

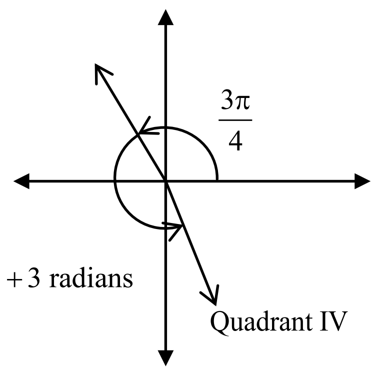
Question 14 T1

An angle in standard position measures $\frac{3\pi}{4}$.

Determine in which quadrant the terminal arm of this angle is located after a rotation of 3 radians.

Justify your answer.

Solution



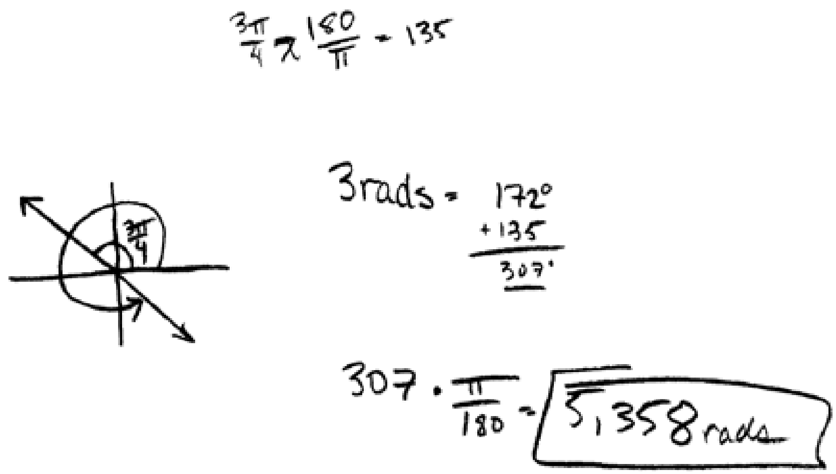
or

The angle $\frac{3\pi}{4}$ terminates in quadrant II. A rotation of 3 radians is almost a half rotation; therefore, the terminal arm is located in quadrant IV.

1 mark for justification

1 mark

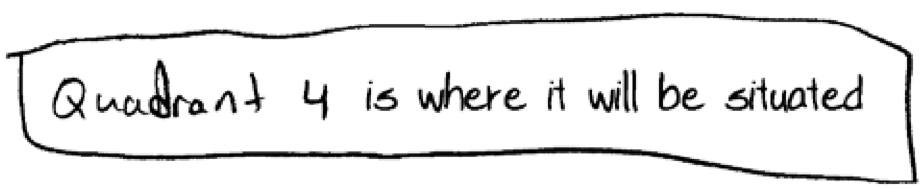
Exemplar 1



1 out of 1

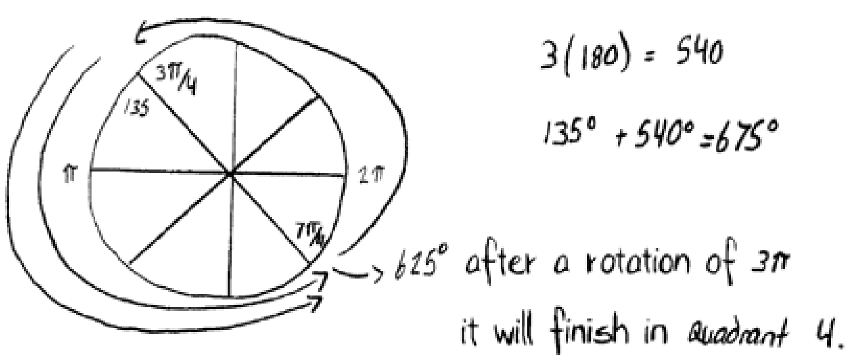
award full marks
E1 (final answer not stated)

Exemplar 2



0 out of 1

Exemplar 3



0 out of 1

Question 15 T6

Prove the following identity for all permissible values of θ.

$$\frac{\sin 2\theta}{1-\cos 2\theta} = \cot \theta$$

Solution

Left-Hand Side	Right-Hand Side
$\frac{\sin 2\theta}{1-\cos 2\theta}$	$\cot \theta$
$\frac{2\sin\theta\cos\theta}{1-\left(1-2\sin^2\theta\right)}$	
$\frac{2\sin\theta\cos\theta}{2\sin^2\theta}$	
$\frac{\cos\theta}{\sin\theta}$	
$\cot\theta$	

1 mark for correct substitution of identities
1 mark for appropriate algebraic strategies
1 mark for logical process to prove the identity

3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$\frac{\sin 2\theta}{1-\cos 2\theta}$	$\cot\theta$
$\frac{2\sin\theta\cos\theta}{1-1-2\sin^2\theta}$	$\downarrow$ $\frac{1}{\tan\theta}$
$\frac{2\sin\theta\cos\theta}{-2\sin^{2}\theta}$	$\downarrow$ $\boxed{\frac{\cos\theta}{\sin\theta}}$
$\boxed{\frac{\cos\theta}{\sin\theta}}$	

2 out of 3

+ 1 mark for correct substitution of identities
+ 1 mark for logical process to prove the identity

Left-Hand Side	Right-Hand Side
$\dfrac{\sin 2\theta}{1-\cos 2\theta}$	
$\dfrac{2\sin\theta\cancel{\cos\theta}}{\cancel{1}-2\cos^2\theta-\cancel{1}}$	
$\dfrac{\cancel{2}\sin\theta}{\cancel{2}\cos\theta}$	
$\dfrac{\sin\theta}{\cos\theta}$	
$\dfrac{\cos\theta}{\sin\theta}$	
$\cot\theta$	$\cot\theta$

LS = RS ✓

1 out of 3

+ 1 mark for correct substitution of identities

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If the range of $y = f(x)$ is $-3 \le y \le 6,$ determine the range of $y = 2f(3x)$.

Solution

$[-6, 12]$

or

$-6 \le y \le 12$

1 mark

Exemplar 1

Range: $(-6, 12)$

1 out of 1

award full marks
E8 (bracket error made when stating range)

Exemplar 2

$f(x)$ $-3 \leq y \leq 6$

$y = 2f(3x)$
↑
would make vertical compression by ½.

$2f(3x)$ $-\frac{3}{2} \leq y \leq 3$.

0 out of 1

Exemplar 3

$-6 \leq y \leq 18$

0 out of 1

Maurice incorrectly solved the equation, $\sin\theta + 1 = 0$, over the interval $\left[0^\circ, 360^\circ\right]$.

$$\sin\theta + 1 = 0$$
$$\sin\theta = -1$$
$$\sin\theta = 270^\circ$$

Describe his error.

Solution

Maurice should have written that θ is equal to 270° not $\sin\theta = 270^\circ$.

1 mark

Exemplar 1

The answer should have been $\theta = 90°$.

0 out of 1

Exemplar 2

The answer was supposed to be $\frac{3\pi}{2}$.

0 out of 1

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Scoring Guidelines for Booklet 2 Questions

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
18	D	R2
19	B	R5
20	B	R7
21	C	T4
22	B	R9
23	D	R12
24	D	T1
25	C	P3
26	A	R14
27	B, C, A, D	R13

Question 18 R2

If $P(3,5)$ is a point on the graph of $y = f(x)$, identify the corresponding point on the graph of $y = f(x-1)+7$.

a) $(2,12)$

b) $(4,-2)$

c) $(2,-2)$

d) $(4,12)$

Question 19 R5

Identify how the graph of $y = 3^x$ is transformed to the graph of $y = 3^{-x}$.

a) reflected over the *x*-axis

b) reflected over the *y*-axis

c) reflected over both the *x*-axis and the *y*-axis

d) reflected over the line $y = x$

Question 20 R7

Identify the equation $\log_a b = c$ in exponential form.

a) $b^c = a$

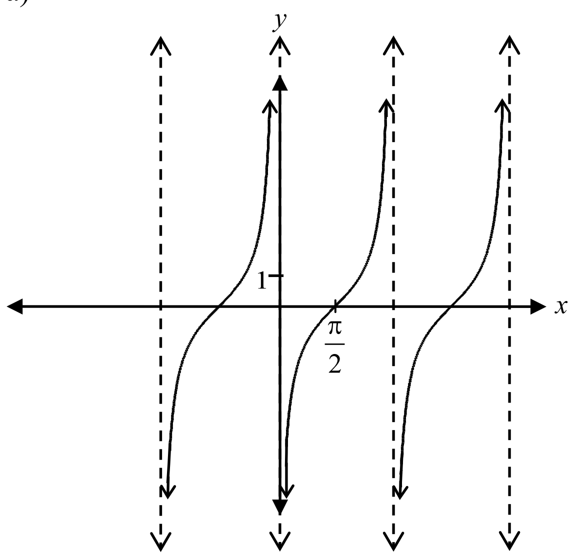
b) $a^c = b$

c) $a^b = c$

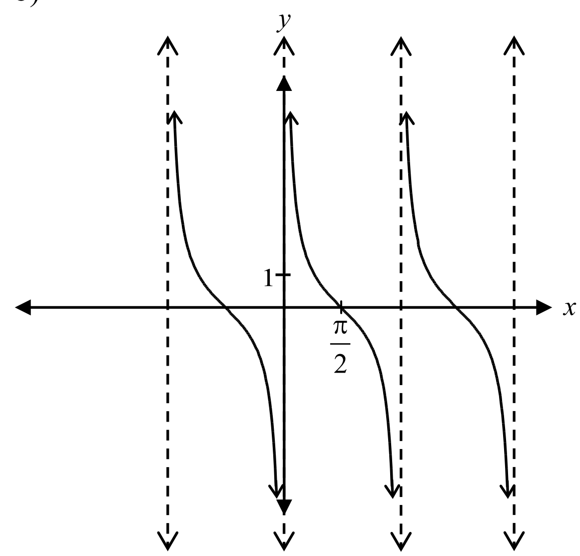
d) $c^a = b$

Identify the graph of $y = \tan x$.

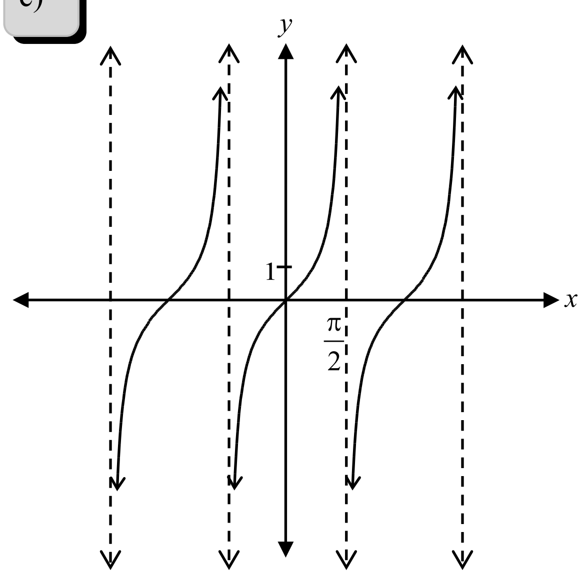
a)



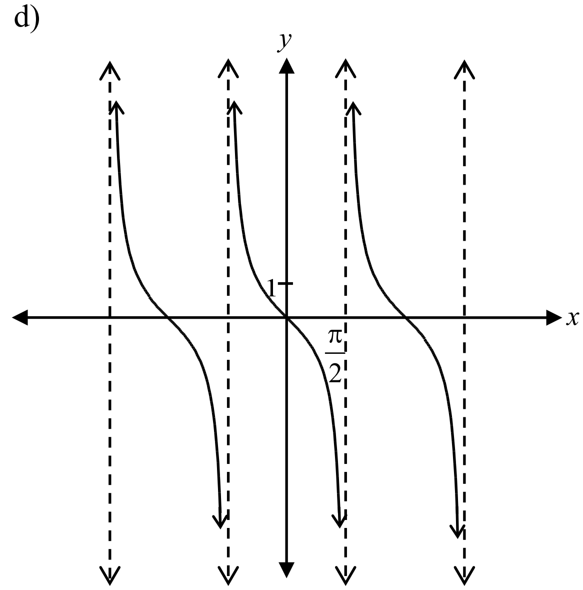
b)



c)



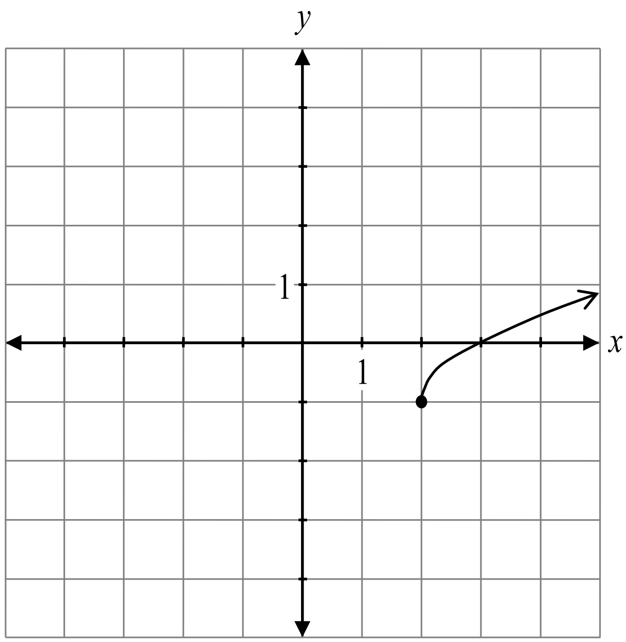
d)



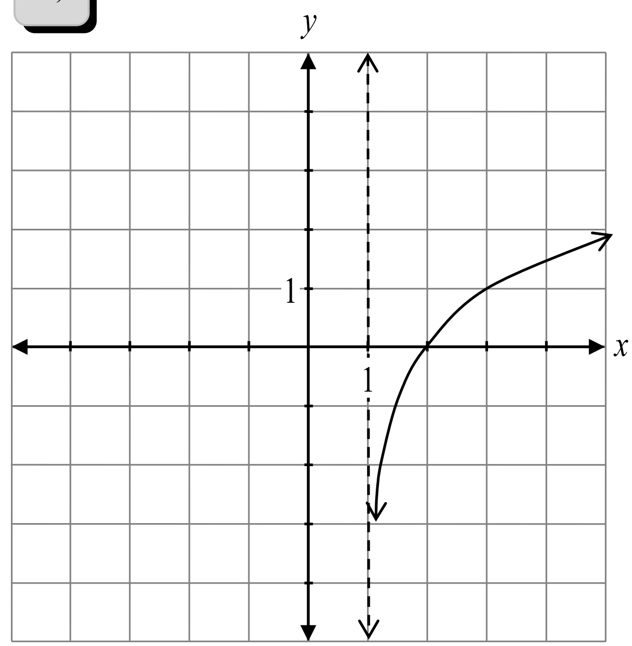
Question 22 R9

Identify which of the following graphs represents a logarithmic function.

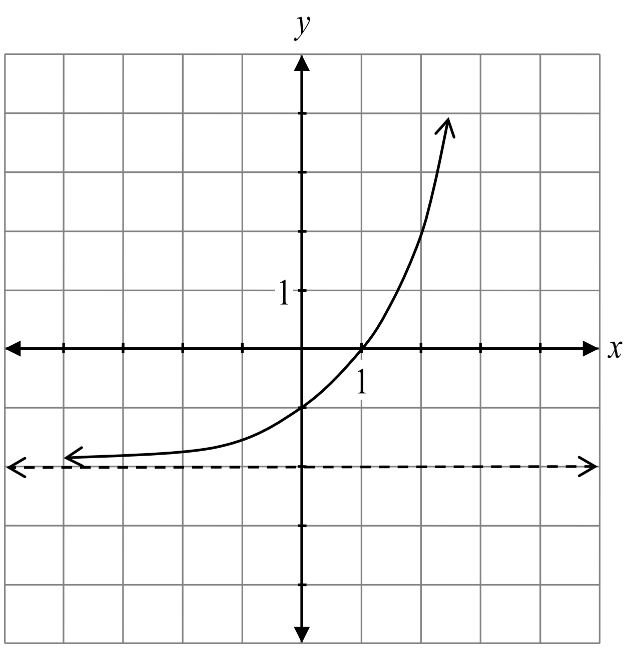
a)



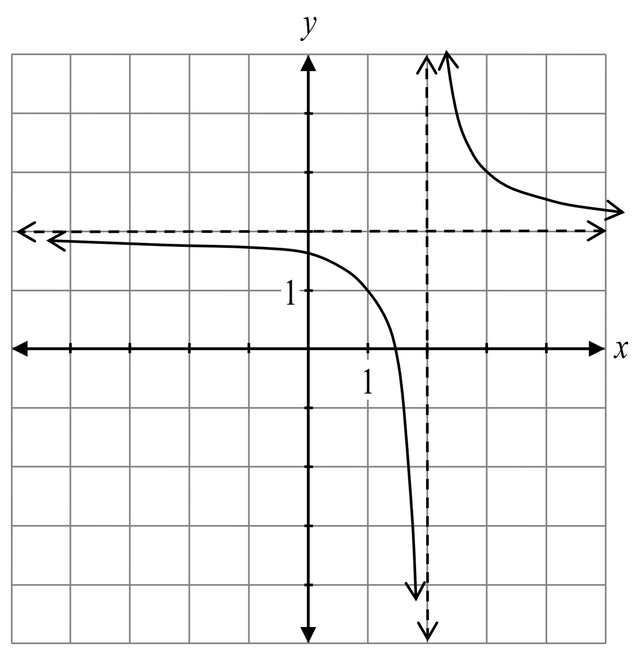
b)



c)



d)



Question 23 R12

If the volume of a box is represented by $V(x) = (x+4)(x+2)(x-1)$, identify a possible value of x.

a) -4

b) -1

c) 1

d) 4

Question 24 T1

Identify a coterminal angle for $\theta = -\frac{\pi}{3}$.

a) $\frac{\pi}{3}$

b) $\frac{4\pi}{3}$

c) $\frac{7\pi}{3}$

d) $\frac{11\pi}{3}$

Question 25 P3

Identify the value of n in the equation ${}_nC_3 = {}_nC_6$.

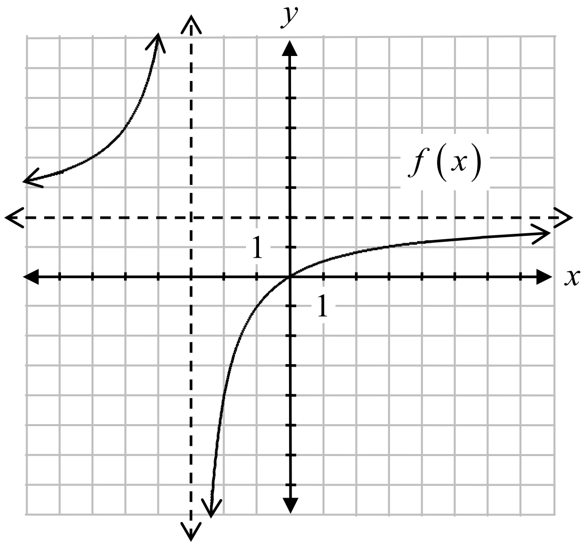
a) 3

b) 6

c) 9

d) 18

Identify the equation of the function, $f(x)$, for the following graph.



a) $f(x) = \dfrac{2x}{x+3}$

b) $f(x) = \dfrac{2}{x+3}$

c) $f(x) = \dfrac{2x^2}{x(x+3)}$

d) $f(x) = \dfrac{3x^2}{x(x+2)}$

Question 27 R13

Match the following radical functions with their graphs.

Solution

Place the appropriate letter in this column.

$f(x) = 2\sqrt{-(x+3)}$ B

$g(x) = -2\sqrt{(x+3)}$ C

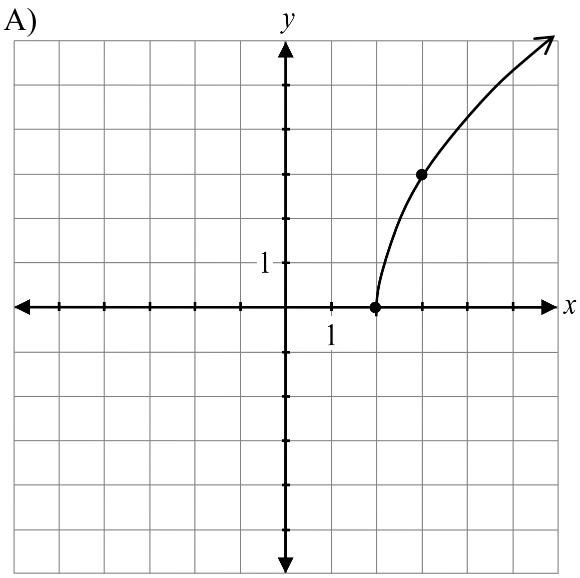
$h(x) = 3\sqrt{(x-2)}$ A

$k(x) = \sqrt{3(x-2)}$ D

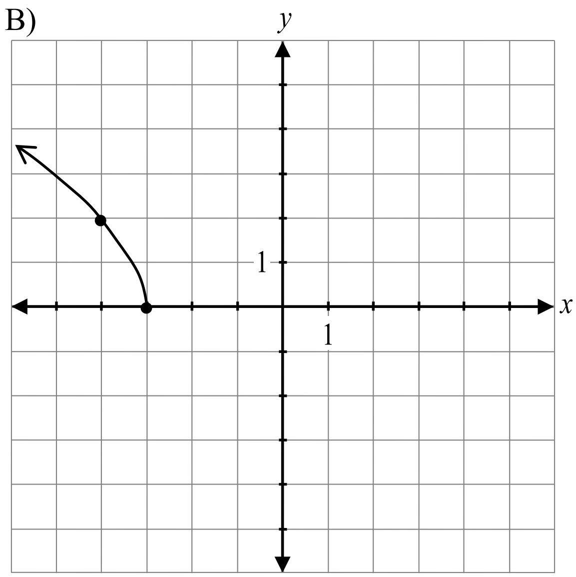
½ mark for each correct answer

2 marks

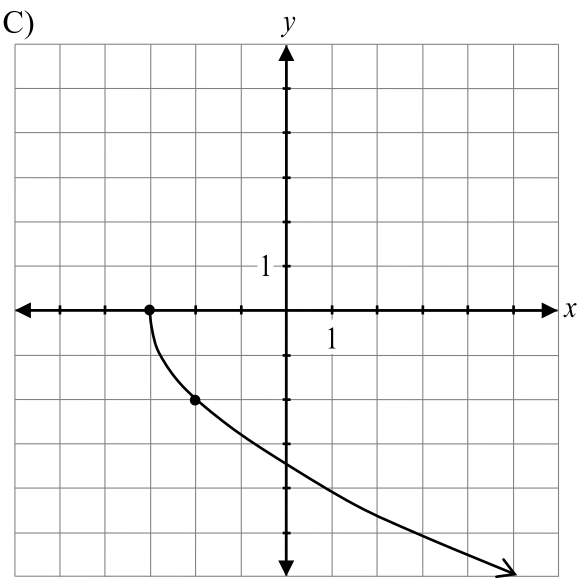
A)



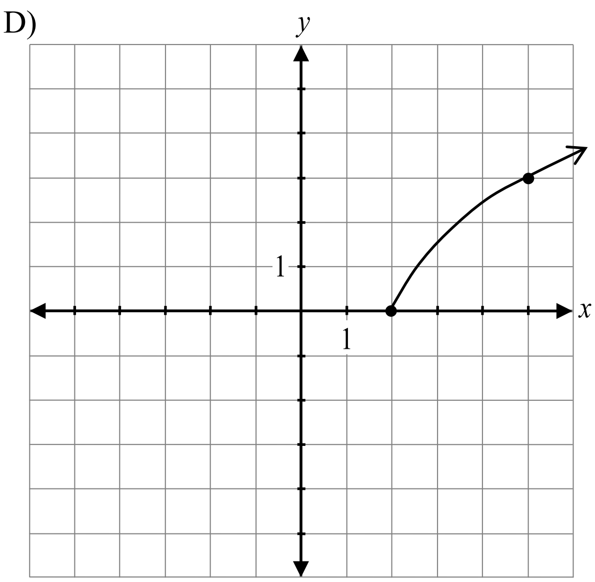
B)



C)



D)



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Express $p(x) = x^3 - 2x^2 - 4x + 8$ as a product of factors.

Solution

$$p(2) = 2^3 - 2(2)^2 - 4(2) + 8$$
$$= 8 - 8 - 8 + 8$$
$$= 0$$

1 mark for identifying one possible value of x

2	1	–2	–4	8
	↓	2	0	– 8
	1	0	–4	0

1 mark for synthetic division (or any equivalent strategy)

$$p(x) = (x-2)\left(x^2 - 4\right)$$

1 mark for product of factors

3 marks

or

$$p(x) = (x-2)(x-2)(x+2)$$

or

$$p(x) = (x-2)^2(x+2)$$

Exemplar 1

$p(x) = x^3 - 2x^2 - 4x + 8$

$P(2) = (2)^3 - 2(2)^2 - 4(2) + 8$

$P(2) = 8 - 8 - 8 + 8$

$P(2) = 0$

$x - 2 \qquad a = 2$

2	1	−2	−4	8
		2	0	−8
	1	0	−4	0

$x^2 - 4$

$(x+2)(x-2)^2$

$x = \pm 2$

2½ out of 3

award full marks
−½ mark for procedural error (solving for the roots)
E2 (changing an equation to an expression in line 6)

Exemplar 2

$P(x) = x^3 - 2x^2 - 4x + 8$

$P(x) = (2)^3 - 2(2)^2 - 4(2) + 8$

$P(x) = 8 - 8 - 8 + 8$

$P(x) = 0$

$\therefore (x - 2)$

2	1	−2	−4	8
	↓	2	0	−8
	1	0	−4	0

$P(x) = x^2 - 4$

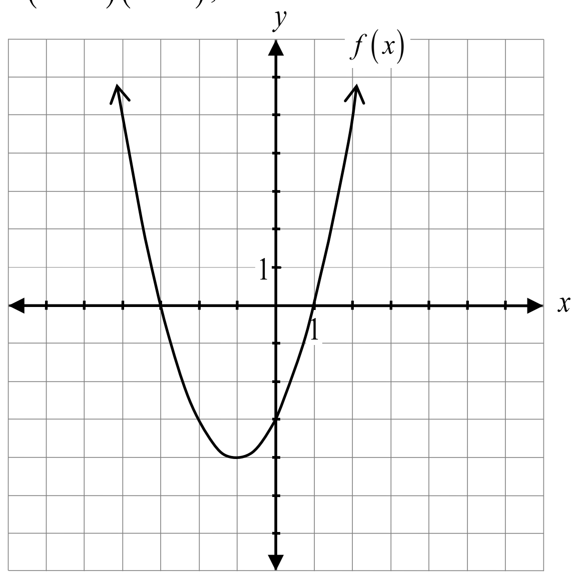
$P(x) = (x+2)(x-2)$

2 out of 3

+ 1 mark for identifying one possible value of x
+ 1 mark for synthetic division
E7 (notation error in lines 2 to 4)

Question 29 R1

Given the graph of $f(x) = (x+3)(x-1)$,

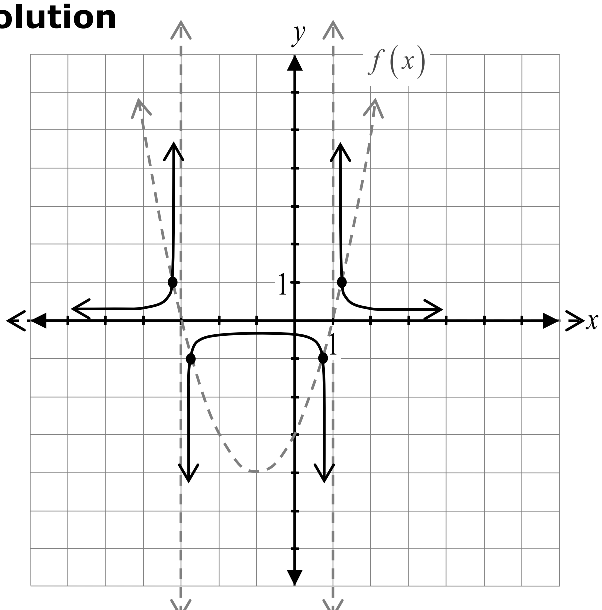


a) sketch the graph of $g(x) = \dfrac{1}{f(x)}$.

b) describe how to sketch the graph of $h(x) = |f(x)|$.

Solution

a)



1 mark for asymptotic behaviour at $x = 1$ and $x = -3$
½ mark for asymptotic behaviour at $y = 0$
½ mark for graph left of $x = -3$
½ mark for graph between $x = -3$ and $x = 1$
½ mark for graph right of $x = 1$

3 marks

b) Change all negative y-values to positive values. The positive y-values do not change.

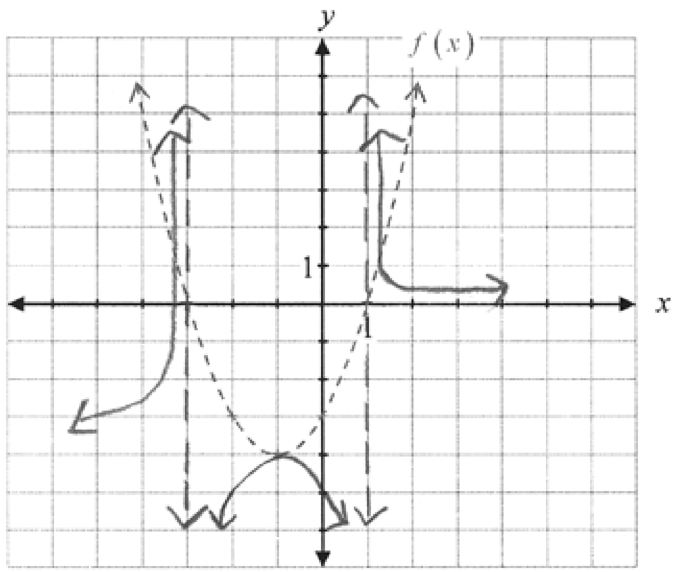
1 mark

Note(s):

- Award 1 mark if only vertical asymptotes at $x = 1$ and $x = -3$ are drawn.

Exemplar 1

a)



1½ out of 3

+ 1 mark for asymptotic behaviour at $x = -3$ and $x = 1$
+ ½ mark for graph right of $x = 1$

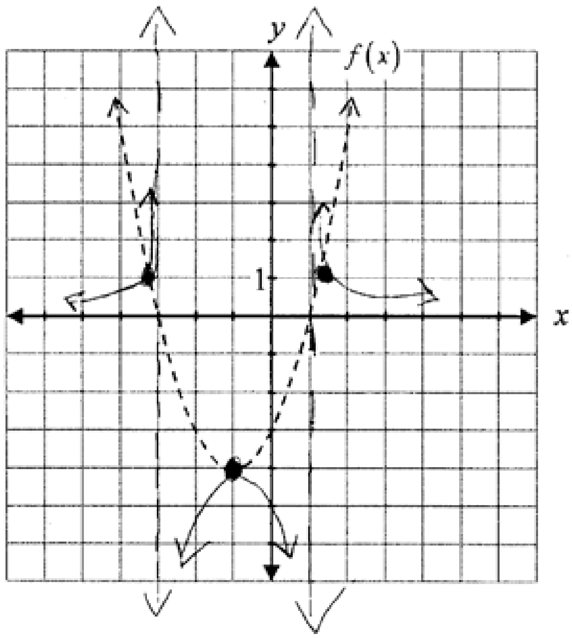
b)

it must be positive
therefore everything under
the x axis must be put
above the axis

1 out of 1

Exemplar 2

a)



2½ out of 3

+ 1 mark for asymptotic behaviour at $x = -3$ and $x = 1$
+ ½ mark for asymptotic behaviour at $y = 0$
+ ½ mark for graph left of $x = -3$
+ ½ mark for graph right of $x = 1$
E10 (asymptotes omitted but still implied)

b)

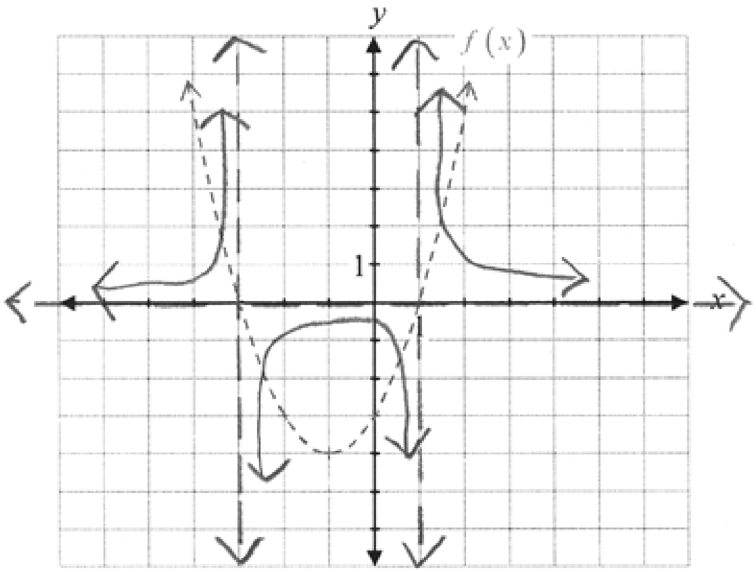
We draw the positive points of f(x) and then we change the negative points (if any) to positive also

½ out of 1

award full marks
−½ mark for lack of clarity in description

Exemplar 3

a)



2½ out of 3

award full marks
− ½ mark for procedural error (correct shape with no correct points)

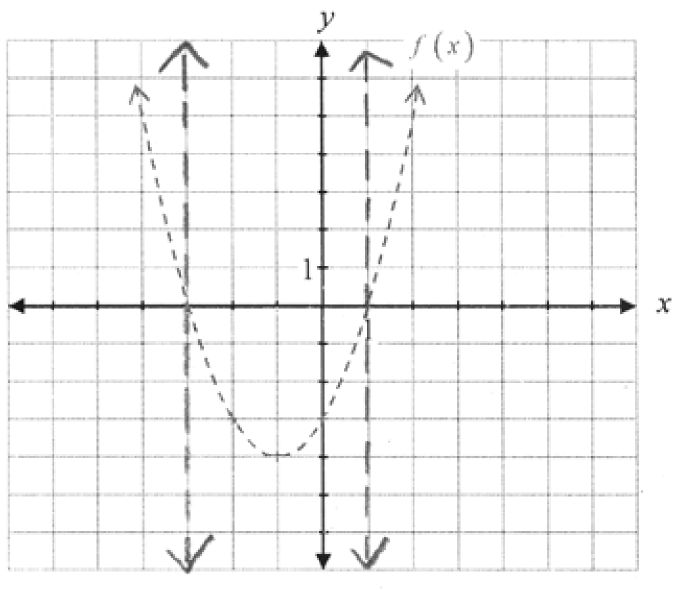
b)

you would take everything below the x axis and reflect it over the x axis.

1 out of 1

Exemplar 4

a)



1 out of 3

+ 1 mark for vertical asymptotes at $x = -3$ and $x = 1$ (see note)

b)

Everything that is negative
becomes positive

½ out of 1

award full marks
− ½ mark for lack of clarity in description

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Question 30 R9

Describe how the value of m in the equation $y = \log_3 (x - m)$, $m \in \mathbb{R}$, affects the asymptote on the graph of $y = \log_3 x$.

Solution

The vertical asymptote is translated horizontally left or right m units from the y-axis.

1 mark

Exemplar 1

The "m" tells where the asympote will be located once solved, it'll tell you the number of where to draw the V.A. to the right.

½ out of 1

award full marks
−½ mark for lack of clarity in description

Exemplar 2

m is the asymptote

½ out of 1

award full marks
−½ mark for lack of clarity in description

Exemplar 3

Depending on the number, the opposite of this number will be where you put the asymptote. For example, if m=1, you put the asymptote at -1.

0 out of 1

Solve algebraically.

$$25^{x} = \left(\frac{1}{5}\right)^{-3x+1}$$

Solution

$$\left(5^{2}\right)^{x} = \left(5^{-1}\right)^{-3x+1}$$ 1 mark for changing to a common base

$$5^{2x} = 5^{3x-1}$$ ½ mark for exponent law

$$2x = 3x - 1$$ ½ mark for equating exponents

$$x = 1$$

2 marks

Exemplar 1

$$(5^2)^x = \frac{1}{5}^{-3x+1}$$

$$2x = -3x + 1$$

$$5x = 1$$

$$\boxed{x = \frac{1}{5}}$$

½ out of 2

+ ½ mark for exponent law
E7 (transcription error in line 1)

Exemplar 2

$$5^{2(x)} = 5^{3x+1}$$

$$2x = 3x + 1$$

$$2x - 3x = 1$$

$$\frac{-1x}{-1} = \frac{1}{-1}$$

$$x = -1$$

1½ out of 2

award full marks
− ½ mark for arithmetic error in line 1

Exemplar 3

$$x \log 25 = (-3x + 1)\log\left(\frac{1}{5}\right)$$

$$x \log 25 = -3x \log\left(\frac{1}{5}\right) + \log\left(\frac{1}{5}\right)$$

$$x \log 25 + 3x \log\left(\frac{1}{5}\right) = \log\left(\frac{1}{5}\right)$$

$$x\left(\log(25) + 3\log\left(\frac{1}{5}\right)\right) = \log\left(\frac{1}{5}\right)$$

$$x = \frac{\log\left(\frac{1}{5}\right)}{\log(25) + 3\log\left(\frac{1}{5}\right)}$$

2 out of 2

award full marks
E1 (final answer not stated)

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Question 32 T5, T6

Solve $\cos 2\theta = 0$, where $\theta \in \mathbb{R}$.

Solution

Method 1

$$1 - 2\sin^2\theta = 0$$ 1 mark for double-angle identity

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \frac{\sqrt{2}}{2} \qquad \sin\theta = \frac{-\sqrt{2}}{2}$$ 1 mark for solving for $\sin\theta$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4} \qquad \theta = \frac{5\pi}{4}, \frac{7\pi}{4}$$ 1 mark for values of θ (½ mark for each branch)

$$\left.\begin{aligned}\theta &= \frac{\pi}{4} + \pi k \\ &= \frac{3\pi}{4} + \pi k\end{aligned}\right\}\text{where } k \in \mathbb{Z} \textbf{ or } \theta = \frac{\pi}{4} + \frac{\pi k}{2}, \text{ where } k \in \mathbb{Z}$$ 1 mark for general solution

4 marks

Method 2

$$2\cos^2\theta - 1 = 0$$ 1 mark for double-angle identity

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \frac{\sqrt{2}}{2} \qquad \cos\theta = \frac{-\sqrt{2}}{2}$$ 1 mark for solving for $\cos\theta$

$$\theta = \frac{\pi}{4}, \frac{7\pi}{4} \qquad \theta = \frac{3\pi}{4}, \frac{5\pi}{4}$$ 1 mark for values of θ (½ mark for each branch)

$$\left.\begin{aligned}\theta &= \frac{\pi}{4} + \pi k \\ &= \frac{3\pi}{4} + \pi k\end{aligned}\right\}\text{where } k \in \mathbb{Z} \textbf{ or } \theta = \frac{\pi}{4} + \frac{\pi k}{2}, \text{ where } k \in \mathbb{Z}$$ 1 mark for general solution

4 marks

Note(s):

- Deduct a maximum of 1 mark if student omits second branch when taking the square root.

Method 3

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ — 2 marks for values of 2θ (1 mark for each value)

$$\left.\begin{aligned} 2\theta &= \frac{\pi}{2} + 2\pi k \\ 2\theta &= \frac{3\pi}{2} + 2\pi k \end{aligned}\right\} \text{where } k \in \mathbb{Z}$$

1 mark for general solution

$$\theta = \frac{\pi}{4} + \pi k$$

$$\theta = \frac{3\pi}{4} + \pi k$$

1 mark for values of θ

4 marks

Exemplar 1

$$\cos 2\theta = 0$$
$$\cos^2\theta - \sin^2\theta = 0$$
$$1 - \sin^2\theta - \sin^2\theta = 0$$
$$-2\sin^2\theta + 1 = 0$$
$$\sqrt{\sin^2\theta} = \sqrt{\frac{1}{2}}$$
$$\sin\theta = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$\sin\theta = \frac{\sqrt{2}}{2}$$

Quad I: $\theta = \frac{\pi}{4}$ Quad II: $\theta = \frac{3\pi}{4}$

$$\left\{\begin{array}{l} \theta = \frac{\pi}{4} \pm 2k\pi \\ \theta = \frac{3\pi}{4} \pm 2k\pi \end{array}\right\} k \in I$$

3 out of 4

+ 1 mark for double-angle identity
+ 1 mark for values of θ
+ 1 mark for general solution

Exemplar 2

let $2\theta = x$

$$\cos x = 0$$
$$\cos x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$
$$\frac{\cos 2\theta}{2} = \frac{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}}{2}$$
$$\cos\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

2 out of 4

+ 2 marks for values of 2θ
+ 1 mark for values of θ

− 1 mark for concept error in lines 3 to 5

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Question 33 R12

Describe a difference between the graphs of $y = f(x)$ and $y = g(x)$.

$$f(x) = -2(x+1)^2(x+3)$$
$$g(x) = 2(x+1)^2(x+3)$$

Solution

The end behaviour is different, because the graph of $f(x)$ falls to the right and the graph of $g(x)$ rises to the right.

or

The y-intercept of $f(x)$ is negative while the y-intercept of $g(x)$ is positive.

or

One graph is a reflection over the x-axis of the other graph.

1 mark

Exemplar 1

Graph f(x) has a reflection because it has a negative coefficient

½ out of 1

award full marks

−½ mark for lack of clarity in description

Exemplar 2

The graph f(x) has a negative coefficient (-2) whereas the graph g(x) has a positive coefficient (2).

0 out of 1

Exemplar 3

f(x) will open down

g(x) will open up

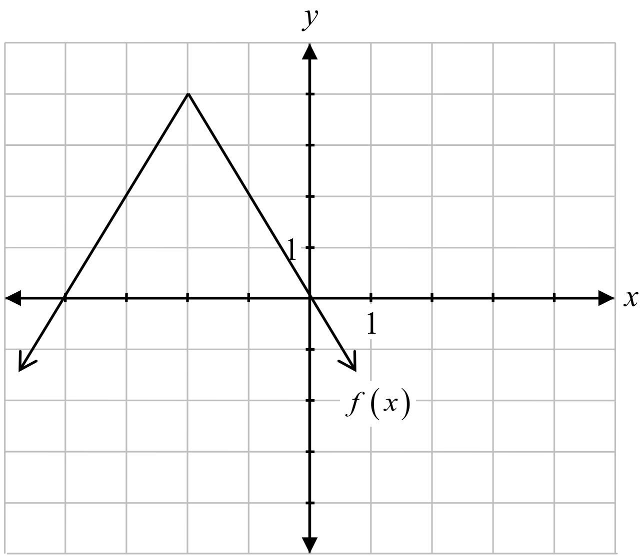
0 out of 1

Exemplar 4

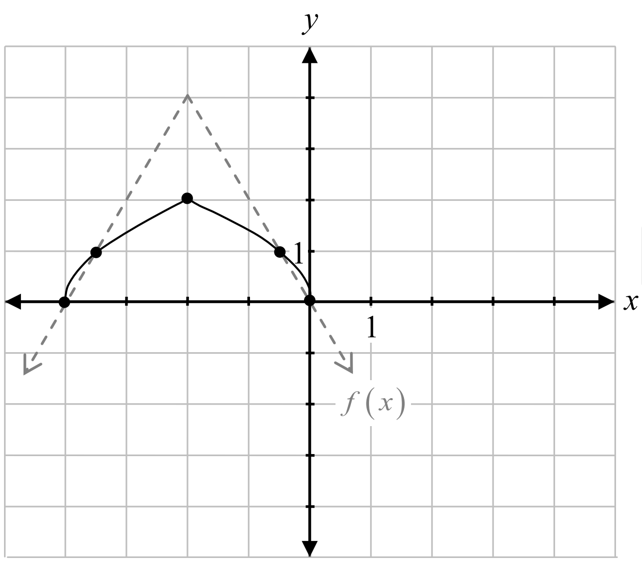
They have different end behaviours.

1 out of 1

Given the graph of $y = f(x)$, sketch the graph of $\sqrt{f(x)}$.



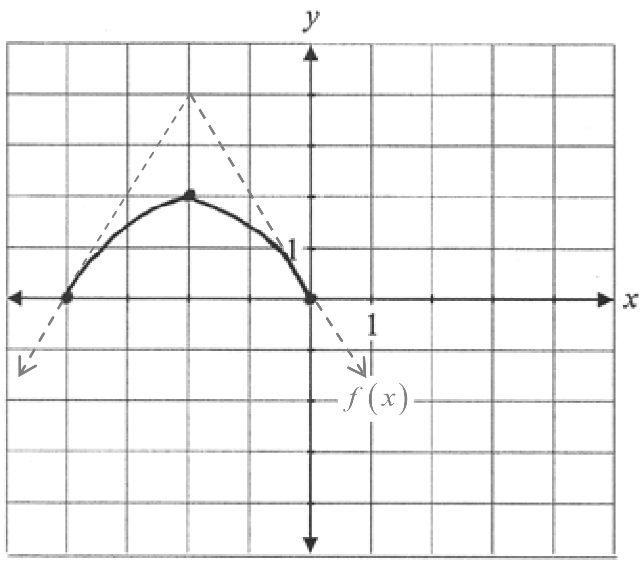
Solution



1 mark for restricted domain
½ mark for shape between both invariant points, $\{0 \leq y \leq 1\}$
½ mark for shape above invariant points, $\{1 \leq y \leq 2\}$

2 marks

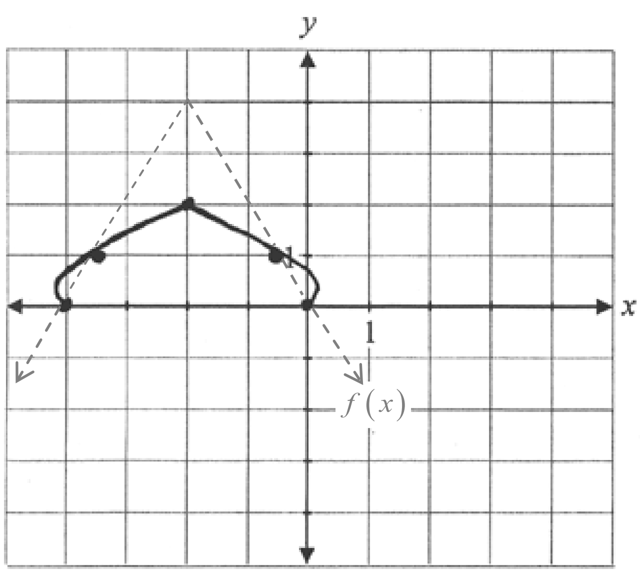
Exemplar 1



1½ out of 2

+ 1 mark for restricted domain
+ ½ mark for shape above invariant points, $\{1 \le y \le 2\}$

Exemplar 2



1½ out of 2

+ 1 mark for restricted domain
+ ½ mark for shape above invariant points, $\{1 \le y \le 2\}$

Question 35 R12

Describe the relationship between the zeros of the function $f(x) = (2x - 1)(x + 3)^2$, the roots of the equation $(2x - 1)(x + 3)^2 = 0$, and the x-intercepts of the graph of $y = f(x)$.

Solution

The zeros, roots, and x-intercepts all have the same values.

1 mark

Exemplar 1

The zeros of the function $f(x) = (2x-1)(x+3)^2$ are $\frac{1}{2}$ and -3. These are the points on the x-axis that the graph touches

½ out of 1

award full mark
−½ mark for lack of clarity in description

Exemplar 2

The zeros, roots and x-intercepts of $(2x-1)(x+3)^2$ are always on the x axis when $(y=0)$.

½ out of 1

award full marks
−½ mark for lack of clarity in description

Exemplar 3

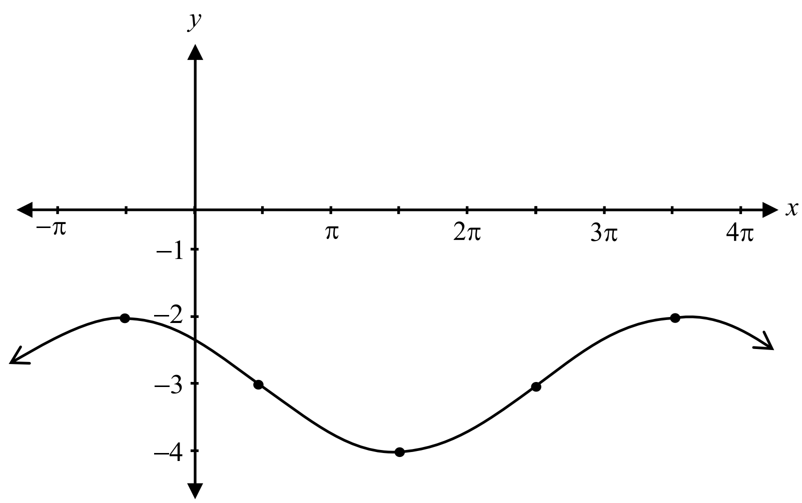
The x's are $1/2$ and -3

0 out of 1

Question 36 T4

Sketch a graph of at least one period of the function $f(x) = \cos\left[\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right] - 3.$

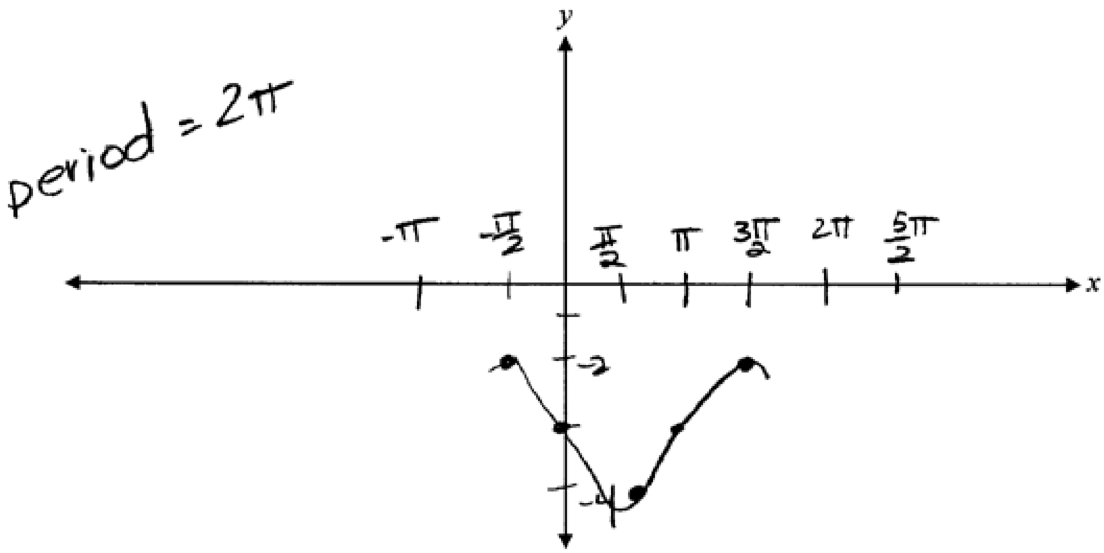
Solution



1 mark for period
1 mark for horizontal translation
1 mark for vertical translation

3 marks

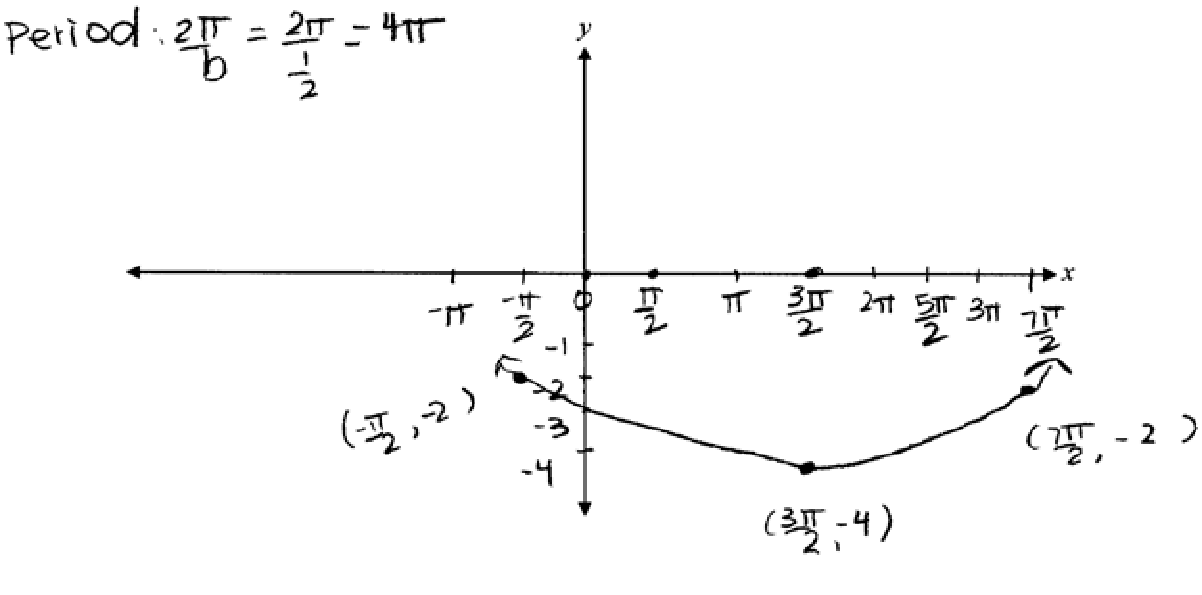
Exemplar 1



2 out of 3

+ 1 mark for horizontal translation
+ 1 mark for vertical translation

Exemplar 2



2½ out of 3

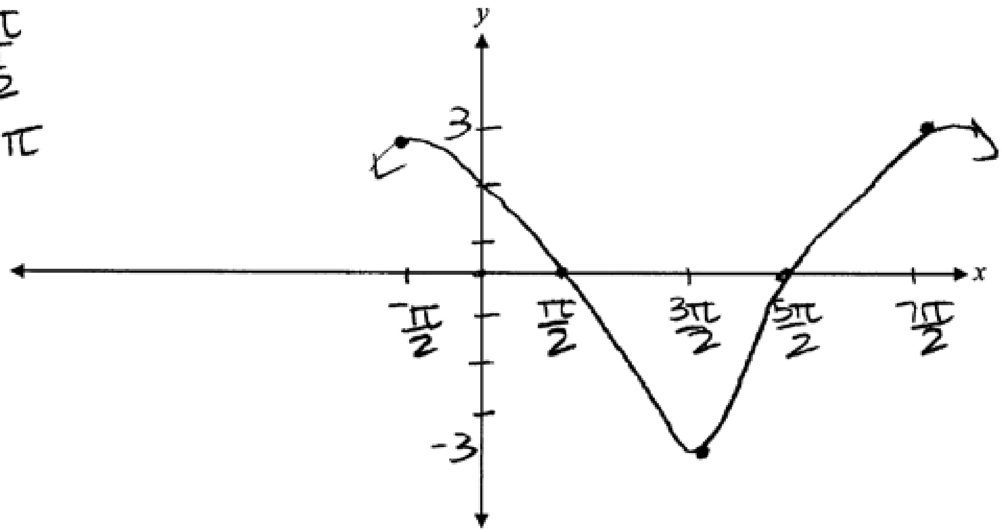
award full marks
−½ mark for incorrect shape of graph
E9 (arrowheads incorrect)

Exemplar 3

$A = 3$

$P = \frac{2\pi}{\frac{1}{2}}$

$= 4\pi$



2 out of 3

+ 1 mark for period
+ 1 mark for horizontal translation

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Question 37 T6

Verify that $\theta = \frac{4\pi}{3}$ is a solution of the equation $4\cos^2\theta - 1 = 0$.

Solution

$$\begin{aligned}\text{Left-hand side} &= 4\cos^2\left(\frac{4\pi}{3}\right) - 1 \\ &= 4\left(\frac{-1}{2}\right)^2 - 1 \\ &= 4\left(\frac{1}{4}\right) - 1 \\ &= 0 \\ &= \text{Right-hand side}\end{aligned}$$

½ mark for value of $\cos\left(\frac{4\pi}{3}\right)$

½ mark for verification

1 mark

Exemplar 1

$$4\cos^2\theta - 1 = 0$$

$$4\cos^2\left(\frac{4\pi}{3}\right) = 0$$

$$4\left(\tfrac{1}{2}\right)^2 - 1 = 0$$

$$\frac{4}{1}\left(\tfrac{1}{4}\right) - 1 = 0$$

$$1 = 1$$

yes, it is a solution to the equation.

½ out of 1

+ ½ mark for verification
E7 (transcription error in line 2)

Exemplar 2

$$4\cos^2\theta - 1 = 0$$

$$= \frac{4\cos^2\theta}{4} = \frac{1}{4}$$

$$= \sqrt{\cos^2\theta} = \sqrt{\tfrac{1}{4}}$$

$$= \cos\theta = \frac{1}{2}$$

$$= \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

Therefore; $\frac{4\pi}{3}$ not a solution, at $\frac{4\pi}{3}$ $\cos\theta \frac{1}{2}$ is negative, and it needs to be positive. $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ are solutions!

½ out of 1

+ ½ mark for verification
E7 (notation error on lines 2 to 5)

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Question 38 R14

Describe how to determine the equation of the horizontal asymptote of a rational function when the degree of the polynomial in the numerator and the degree of the polynomial in the denominator are equal.

Solution

The horizontal asymptote is $y = \frac{a}{b}$ where a is the leading coefficient in the numerator and b is the leading coefficient in the denominator.

or

Use polynomial division to divide the numerator by the denominator. The equation of the horizontal asymptote will have the same value as the quotient.

1 mark

Exemplar 1

Ex: $y = \frac{2x}{x+1}$ Horizontal Asymptote : $y = \frac{2}{1}$

$y = 2$

0 out of 1

Exemplar 2

In this situation, you divide the coefficient of the numerator by the coefficient of the denominator. That number is where your horizontal asymptote will lie.

½ out of 1

award full marks
−½ mark for lack of clarity in description

Exemplar 3

If your degrees are eqaul on top & bottom your leading coefficent tells you your asymptote

½ out of 1

award full marks
−½ mark for lack of clarity in description

Exemplar 4

If the degrees are equal, the asymptote is $y=1$.

0 out of 1

Question 39 T3

Evaluate.

$$\frac{\cot\left(-\frac{5\pi}{6}\right)}{\sin\left(\frac{17\pi}{3}\right)}$$

Solution

$$\frac{\sqrt{3}}{-\frac{\sqrt{3}}{2}}$$

$$\left(\sqrt{3}\right)\left(-\frac{2}{\sqrt{3}}\right)$$

$$-2$$

1 mark for $\cot\left(-\frac{5\pi}{6}\right)$ (½ mark for value, ½ mark for quadrant)

1 mark for $\sin\left(\frac{17\pi}{3}\right)$ (½ mark for value, ½ mark for quadrant)

2 marks

Exemplar 1

$$= \frac{(\sqrt{3})}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{\sqrt{3}}{1} \times \frac{2}{\sqrt{3}} = 2$$

1½ out of 2

+ 1 mark for $\cot\left(\frac{-5\pi}{6}\right)$

+ ½ mark for value of $\sin\left(\frac{17\pi}{3}\right)$

Exemplar 2

$$= \frac{\sqrt{3}}{-\frac{1}{2}}$$

$$\boxed{= -2\sqrt{3}}$$

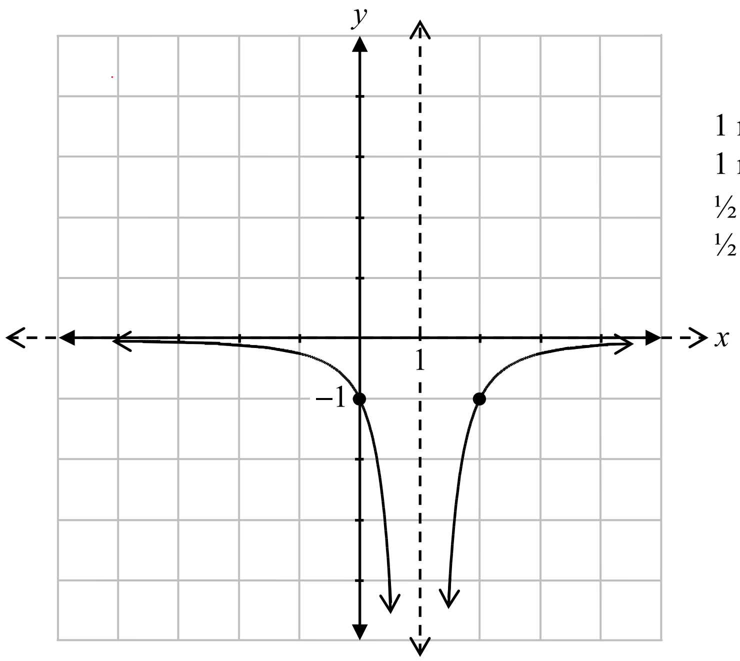
1½ out of 2

+ 1 mark for $\cot\left(\frac{-5\pi}{6}\right)$

+ ½ mark for quadrant of $\sin\left(\frac{17\pi}{3}\right)$

Sketch the graph of the function $f(x) = \dfrac{-1}{(x-1)^2}$ and determine the range.

Solution



1 mark for asymptotic behaviour at $x = 1$
1 mark for asymptotic behaviour at $y = 0$
½ mark for graph left of $x = 1$
½ mark for graph right of $x = 1$

Range: $\{y \in \mathbb{R} \mid y < 0\}$

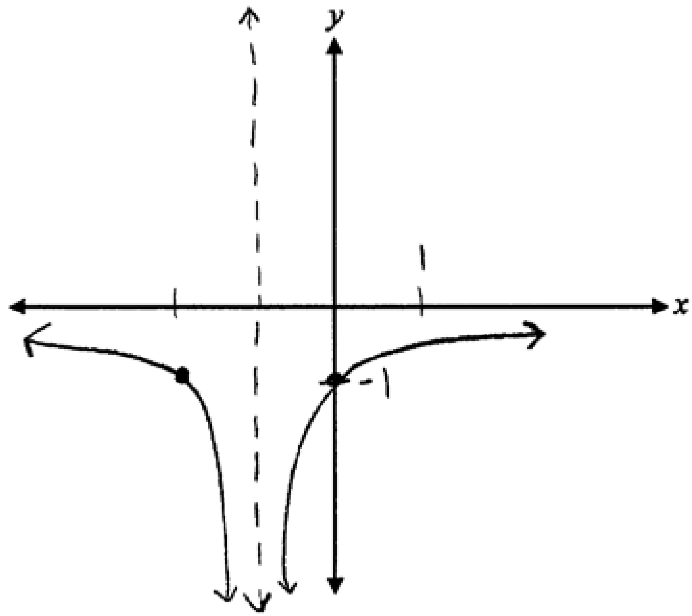
or

Range: $(-\infty, 0)$

1 mark for range (consistent with graph)

4 marks

Exemplar 1

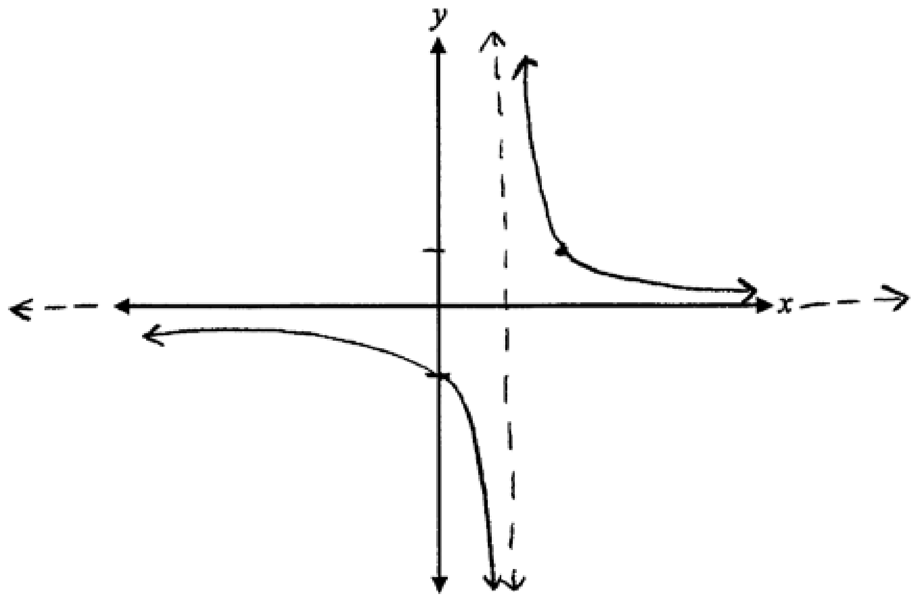


Range: $y \leq 0$

3 out of 4

+ 1 mark for asymptotic behaviour at $y = 0$
+ ½ mark for graph left of asymptote
+ ½ mark for graph right of asymptote
+ 1 mark for range
E8 (bracket error made when stating range)
E10 (asymptotes omitted but still implied)

Exemplar 2



Range: $y \neq 0$

3½ out of 4

+ 1 mark for asymptotic behaviour at $x = 1$
+ 1 mark for asymptotic behaviour at $y = 0$
+ ½ mark for graph left of $x = 1$
+ 1 mark for range (consistent with graph)
E9 (scale values on axes not indicated)

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Given $f(x) = \sqrt{x-2}$ and $g(x) = x^2 + 1$,

a) determine $g(f(x))$.

b) explain why the domain of $g(f(x))$ is restricted.

Solution

a) $g(f(x)) = \left(\sqrt{x-2}\right)^2 + 1$ 1 mark for composition

$= x - 2 + 1,\ x \geq 2$

$= x - 1,\ x \geq 2$

1 mark

b) The domain of $g(f(x))$ must be restricted because the domain of $f(x)$ is restricted.

or

The value of the radicand must be positive.

1 mark

Exemplar 1

a)

$g(x) = (\sqrt{x-2})^2 + 1$

$g(f(x)) =$ $x - 2 + 1$

---

1 out of 1

award full marks
E7 (notation error in line 1)

b)

Because the graph stopped at x=2 because of the domain of f(x)

---

1 out of 1

Exemplar 2

a)

$$g(f(x)) = (\sqrt{x-2})^2 + 1$$
$$= x - 2 + 1$$
$$= x - 1$$
$$\boxed{1 = x}$$

½ out of 1

award full marks
−½ mark for procedural error (solving for x)

b)

The domain of $f(x)$ is not $x \in R$ so the domain of $g(f(x))$ cannot be as well

1 out of 1

Exemplar 3

a)

$g(f(x)) = (\sqrt{x-2})^2 + 1$

1 out of 1

b)

Because a square root cannot be negative

½ out of 1

award full marks
− ½ mark for terminology error in explanation

Question 42 R7, R10

Solve algebraically.

$$2\log_a 3 + \log_a 4 = 2, \text{ where } a > 0$$

Solution

$$\log_a \left(3^2 \bullet 4\right) = 2$$

$$\log_a 36 = 2$$

$$a^2 = 36$$

$$a = 6$$

1 mark for power law
1 mark for product law

1 mark for exponential form

3 marks

Exemplar 1

$$\frac{\not{2}\log_a(3\cdot 4)}{2} = \frac{2}{2}$$

$$\log_a 12 = 1$$

$$a^1 = 12$$

$$\boxed{a = 12}$$

2 out of 3

+ 1 mark for product law
+ 1 mark for exponential form

Exemplar 2

$$\text{Log}_a\left[(3^2)(4)\right] = 2$$

$$\text{Log}_a\ 32 = 2$$

$$a^2 = 32$$

$$a = \sqrt{32}$$

2½ out of 3

award full marks
–½ mark for arithmetic error in line 2

Exemplar 3

$$\log_a(3^2 \cdot 4) = 2$$

$$\log_a(9 \cdot 4) = 2$$

$$\log_a(36) = 2$$

$$\sqrt{a^2} = \sqrt{36}$$

$$a = \mp 6$$

3 out of 3

award full marks
E8 (answer outside the given domain)

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Question 43 T5

Solve $\sec\theta + 2 = 0$ over the interval $[0, 2\pi]$.

Solution

$$\sec\theta + 2 = 0$$
$$\sec\theta = -2$$
$$\cos\theta = -\frac{1}{2}$$

1 mark for reciprocal

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

1 mark for values of θ (½ mark for each value)

2 marks

Exemplar 1

$\sec\theta = -2$

then $\cos\theta = -\frac{1}{2}$

$\theta: = \frac{5\pi}{6}, \frac{7\pi}{6}$

Solution: $\theta = \frac{5\pi}{6}$

$\theta = \frac{7\pi}{6}$

1½ out of 2

+ 1 mark for reciprocal

+ ½ mark for consistent value of θ ($\frac{7\pi}{6}$ is consistent with the reference angle of $\frac{5\pi}{6}$)

Exemplar 2

$\sec\theta + 2 = 0$

$\sec\theta = -2$

$\cos\theta = -\frac{1}{2}$

$\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

1 out of 2

+ 1 mark for reciprocal

Exemplar 3

$\sec\theta + 2 = 0$

$\sec\theta = -2$

no solution

0 out of 2

Determine the x-intercept of the graph of $f(x) = e^x - 1$.

Solution

Method 1

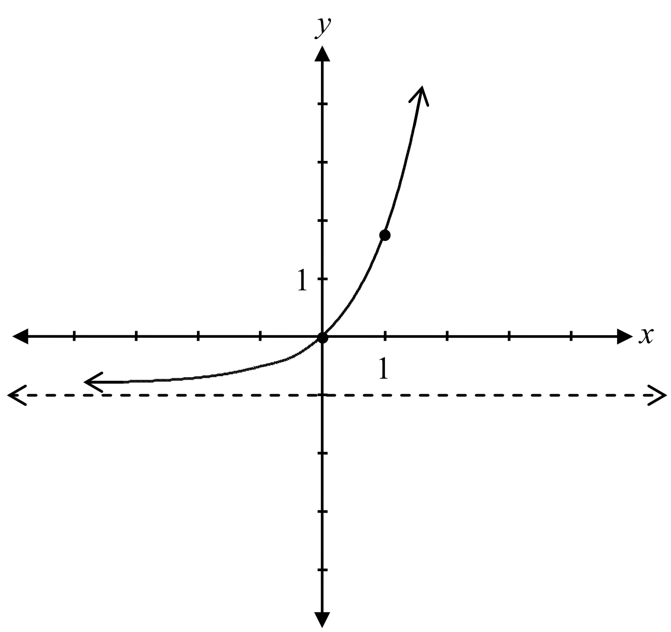
$0 = e^x - 1$ ½ mark for substitution

$1 = e^x$

$x = 0$ ½ mark for solving for x

1 mark

Method 2



$\therefore x = 0$ **1 mark**

Exemplar 1

$0 = e^x - 1$

$\ln 1 = \ln e^x$

$\ln 1 = x$ ~~ln e~~ 1

$\ln 1 = x$

1 out of 1

award full marks
E1 (final answer not stated)

Exemplar 2

$y = e^{(0)} - 1$

$y = 0 - 1$

$y = -1$

0 out of 1

Exemplar 3

$f(x) = e^x - 1$

$0 = e^x - 1$

$\boxed{1 = e^x}$

½ out of 1

+ ½ mark for substitution

Given the 5th row of Pascal's triangle, determine the values of the next row.

1 4 6 4 1

Solution

1 5 10 10 5 1

1 mark

Exemplar 1

5 10 10 5

0 out of 1

Exemplar 2

1 5 24 24 5 1

0 out of 1

Question 46 R7, R8

Evaluate.

$$\log_2 80 - \log_2 10$$

Solution

$\log_2\left(\frac{80}{10}\right)$	1 mark for quotient law
$\log_2 8$	
3	1 mark for evaluating the logarithm

2 marks

Exemplar 1

$$\log_2\left(\frac{80}{10}\right)$$

$$\log_2 8$$

1 out of 2

+ 1 mark for quotient law

Exemplar 2

$$\log_2\left(\frac{80}{10}\right)$$

$$\log_2 8$$

$$2^x = 8$$

$$2^x = 2^3$$

$$\boxed{x = 3}$$

2 out of 2

award full marks
E3 (variables introduced without being defined in line 3)

State the amplitude of $f\left(x\right) = -2\sin\left(x - \pi\right) - 1$.

Solution

2 **1 mark**

Exemplar 1

-2

0 out of 1

Question 48 — T3, T6

Determine the exact value of $\cos 15^\circ$.

Solution

$$
\begin{aligned}
\cos\left(15^\circ\right) &= \cos\left(60^\circ - 45^\circ\right) \\
&= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ \\
&= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\
&= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \\
&= \frac{\sqrt{2}+\sqrt{6}}{4}
\end{aligned}
$$

1 mark for substitution into correct identity

2 marks for values (½ mark for each exact value)

3 marks

Note(s):

- Other combinations are possible.

Exemplar 1

$$= \cos 60^\circ - \cos 45^\circ$$

$$\frac{1}{2} - \frac{\sqrt{2}}{2}$$

$$= \frac{1-\sqrt{2}}{2}$$

1 out of 3

+ ½ mark for value of $\cos 60^\circ$
+ ½ mark for value of $\cos 45^\circ$

Exemplar 2

$$(\cos 45 - 30) = \cos 45 \cos 30 + \sin 45 \sin 30$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{2\sqrt{2} + \sqrt{3}}{2}$$

2½ out of 3

award full marks
−½ mark for arithmetic error in line 3
E4 (missing brackets but still implied in line 1)

Exemplar 3

$(\cos 45° - \cos 30°)$

$\cos 45: \sqrt{2}/2$
$\sin 45$

$\cos 30\ \sqrt{3}/2$
$\sin 30: 1/2$

$(\cos 15°) = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{2}{4} + \frac{\sqrt{3}}{4} =$

$\cos 15° = \frac{2\sqrt{3}}{4}$

1 out of 3

+ 2 marks for values
− ½ mark for procedural error in line 1
− ½ mark for arithmetic error in line 4
E2 (changing an equation to an expression in lines 1 and 3)

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Question 49 R1

Given $f(x) = x^2 + 5x + 6$, $g(x) = x + 3$, and $h(x) = f(x) - g(x)$,

a) determine $h(x)$.

b) sketch the graph of $y = h(x)$.

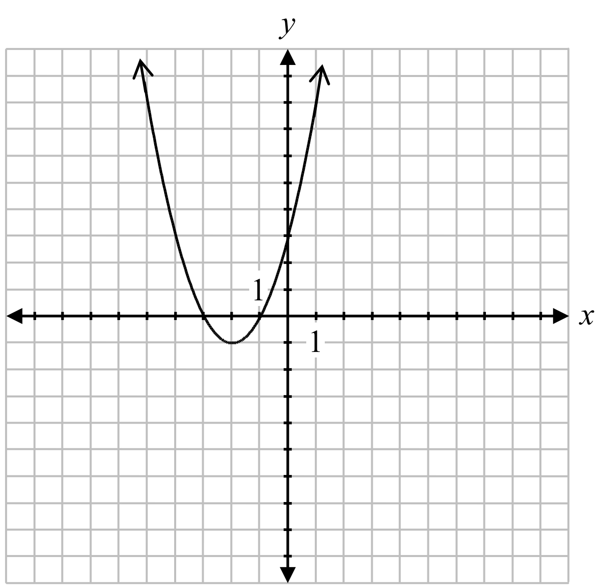
Solution

a)

$$f(x) - g(x) = x^2 + 5x + 6 - (x + 3)$$
$$= x^2 + 4x + 3$$

1 mark

b)



1 mark for graph consistent with a)

1 mark

Exemplar 1

a)

$f(x - g(x))$

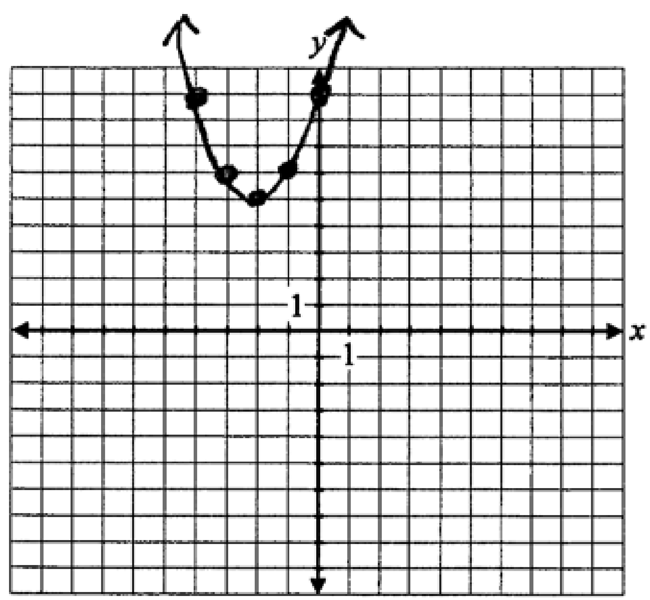
$x^2 + 5x + 6 - x + 3$

$h(x) =$ $\boxed{x^2 + 4x + 9}$

½ out of 1

award full marks
− ½ mark for arithmetic error
E7 (notation error in line 1)

b)



x	y
−3	6
−2	5
−1	6
0	9

1 out of 1

graph consistent with answer in a)

Exemplar 2

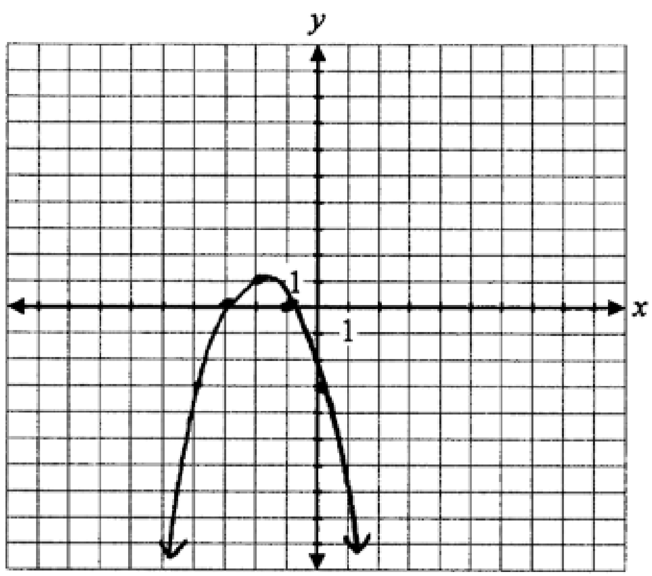
a)

$f(x) - g(x) = (x + 3) - (x^2 + 5x + 6)$

$h(x) =$ $= -x^2 - 4x - 3$

0 out of 1

b)



1 out of 1

graph consistent with error in a)

Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ " $\sin x^2$ " written instead of " $\sin^2 x$ " ▪ missing brackets but still implied
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: ______________________________

Date marked: ______________________________

Booklet No.: ______________________________

Problem(s) noted: ______________________________

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Question(s) affected: ______________________________

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Action taken or rationale for assigning marks: ______________________________

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Follow-up: ____

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Decision: ____

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Marker's Signature: ____

Principal's Signature: ____

For Department Use Only—After Marking Complete

Consultant: ____

Date: ____

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
6	R4, R5	2
12	R6	2
16	R3	1
18	R2	1
19	R5	1
29a)	R1	3
29b)	R1	1
41a)	R1	1
41b)	R1	1
49a)	R1	1
49b)	R1	1
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
1	T1	2
7	T2, T3	2
14	T1	1
21	T4	1
24	T1	1
36	T4	3
39	T3	2
47	T4	1
48	T3	2
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
2	P3	2
4	P4	3
8	P2	3
11	P1	1
25	P3	1
45	P4	1
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
10	R12	2
23	R12	1
28	R11	3
33	R12	1
35	R12	1

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
5	T5	3
15	T6	3
17	T5	1
32	T5, T6	4
37	T6	1
43	T5	2
48	T6	1
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
3	R10	2
20	R7	1
22	R9	1
30	R9	1
31	R10	2
42	R7, R10	3
44	R9	1
46	R7, R8	2
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
9	R13	2
13	R13	1
26	R14	1
27	R13	2
34	R13	2
38	R14	1
40	R14	4